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一般不确定转移速率下 Markov 切换系统的弹性控制器

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摘 要: 研究了一类随机时滞 Markov 切换系统的弹性控制器设计问题. 该系统的转移速率是一般不确定的,比完全已知速率和不完全已知速率更具有一般性. 针对此类 Markov 切换系统,充分考虑一般不确定转移速率矩阵中各元素之间的特性,通过构建一个较为新颖的模态依赖型 Lyapunov-Krasovskii 泛函,设计了弹性控制器以确保闭环系统随机稳定. 并且,通过求解一组线性矩阵不等式得到控制器增益矩阵. 最后,利用一个数值算例验证了所得结果的有效性.

关 键 词: Markov 切换系统;模态依赖;一般不确定转移速率;随机稳定;弹性控制器
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Resilient Controller for Markov Switching Systems Under Generally Uncertain Transition Rate

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Abstract: The problem of resilient controller design for stochastic time-delayed Markov switching systems was investigated. Transition rate of the system is generally uncertain, which is more general than the completely known rate and the partly known rate. By full considering features between each element in the generally uncertain transition matrix, a mode-dependent Lyapunov-Krasovskii functional was established, and a resilient controller was designed to ensure that the closed-loop system was stochastically stable for the Markov switching systems. A set of linear matrix inequalities (LMIs) was solved to get controller gain matrix. Finally, a numerical example was given to demonstrate the effectiveness of the results.

Key words: Markov switching systems; mode-dependent; generally uncertain transition rate; stochastic stability; resilient controller

Markov 切换系统作为一种特殊的切换系统,已被广泛研究,例如经济系统、网络控制系统、容错控制系统等都可以用 Markov 切换系统建模描述[1-4]. 转移速率(关键性因素)决定了 Markov 切换系统的性能. 近些年,针对转移速率问题的研究,主要集中在转移速率完全已知,或者转移速率部分已知的情况[5-9],但由于控制系统的复杂性,获得准确的转移速率代价高昂并且几乎是不可能的. 而一般不确定转移速率涵盖了转移速率部分未知和转移速率不确定两大内容,因此,此类 Markov 切换系统的研究更具有实际意义[10-13].

同时,由于随机扰动和时滞在各类动力系统中的客观存在,随机时滞微分方程作为实用意义很强的一类系统模型也被广泛研究[14]. 进而,随机时滞 Markov 切换系统也取得了许多有意义的成果[15-17].

控制器参数的微小摄动通常会大幅降低闭环系统的性能,但是这种摄动是不可避免的,因此,

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弹性控制器在工业过程中起到十分重要的作用,其设计问题引起了许多学者的关注[18-19].

目前,对于一般转移速率下随机时滞 Markov 切换系统的弹性控制的文献还很少见,本文针对这类系统,构造了模态依赖型的 Lyapunov - Krasovskii 泛函,结合自由权矩阵得到了保守性较低的闭环系统的随机稳定的充分条件.在此基础上,设计了弹性控制器以确保闭环系统的稳定性.最后,通过数值仿真验证了所得结果的有效性及优势.

1 系统描述

考虑如下随机时滞 Markov 切换系统:

$$\begin{aligned}\mathrm{d}\boldsymbol{x}(t)=&[\boldsymbol{A}(g_t)\boldsymbol{x}(t)+\boldsymbol{A}_{\mathrm{d}}(g_t)\boldsymbol{x}(t-\tau(t))+\\&\boldsymbol{B}(g_t)\boldsymbol{u}(t)]\mathrm{d}t+[\boldsymbol{W}(g_t)\boldsymbol{x}(t)+\\&\boldsymbol{W}_{\mathrm{d}}(g_t)\boldsymbol{x}(t-\tau(t))]\mathrm{d}w(t)\ ,\\&\boldsymbol{x}(t+\theta)=\boldsymbol{\varphi}(\theta),\theta\in[-t,0].\end{aligned}\qquad(1)$$

式中:$\boldsymbol{x}(t)\in\mathbf{R}^n$ 是状态向量;$\boldsymbol{u}(t)\in\mathbf{R}^m$ 是控制输入;$w(t)$ 是标准维纳过程;时变时滞 $\tau(t)$ 满足 $0\leqslant\tau(t)\leqslant\tau,\dot{\tau}(t)\leqslant h<1$;初始函数 $\boldsymbol{\varphi}(\theta)$ 定义在区间 $[-\tau,0]$ 上;g_t 为有限集 $S=\{1,2,\cdots,N\}$ 中取值的连续 Markov 过程.从 t 时刻模态 i 到 $t+\Delta t$ 时刻模态 j 的转移概率为

$$P\{g_{t+\Delta t}=j|g_t=i\}=\begin{cases}\hat{\pi}_{ij}\Delta t+o(\Delta t), & i\neq j;\\1+\hat{\pi}_{ii}\Delta t+o(\Delta t), & i=j.\end{cases}$$

式中:$\Delta t\geqslant0,\lim\limits_{\Delta t\geqslant0}(o(\Delta t)/\Delta t)=0$;$\hat{\pi}_{ij}$ 为系统转移速率,并且当 $i\neq j$ 时满足

$$\hat{\pi}_{ij}\geqslant0,\sum_{j=1,i\neq j}^{N}\hat{\pi}_{ij}=-\hat{\pi}_{ii}\ .$$

本文考虑一般不确定转移速率:

$$\hat{\pi}_{ij}=\pi_{ij}+\Delta\pi_{ij}.$$

式中:π_{ij} 为估计值;$\Delta\pi_{ij}\in[-\lambda_{ij},\lambda_{ij}]$,$(\lambda_{ij}>0)$ 为 $\hat{\pi}_{ij}$ 的误差并且分别满足

$$\sum_{j=1,i\neq j}^{N}\pi_{ij}=-\pi_{ii},\sum_{j=1,i\neq j}^{N}\Delta\pi_{ij}=-\Delta\pi_{ii},|\pi_{ij}|>|\lambda_{ij}|.$$

式中,π_{ij} 和 λ_{ij} 是已知先验的.假设三模态的一般不确定转移速率矩阵为

$$\boldsymbol{\Pi}=\begin{bmatrix}? & ? & \pi_{13}+\Delta\pi_{13}\\? & \pi_{22}+\Delta\pi_{22} & ?\\\pi_{31}+\Delta\pi_{31} & ? & \pi_{33}+\Delta\pi_{33}\end{bmatrix}.$$

其中,未知元素用"?"表示.对于 $\forall i\in S,S^i=S_{\mathrm{k}}^i+S_{\mathrm{uk}}^i$,这里 $S_{\mathrm{k}}^i\triangleq\{j:\pi_{ij}$ 是已知的$,j\in S\}$,$S_{\mathrm{uk}}^i\triangleq\{j:\pi_{ij}$ 是未知的$,j\in S\}$.如果 $S^i\neq\varnothing$,可以描述为 $S_{\mathrm{k}}^i\triangleq\{k_1^i,k_2^i,\cdots,k_m^i\},1\leqslant m\leqslant N$.其中 $k_m^i\in S$ 代表矩阵 $\boldsymbol{\Pi}$ 第 i 行中序号为 k_m^i 的第 m 个已知元素.方便起见,定义矩阵中的符号为 $g_t=i$.

本文设计弹性状态反馈控制器为

$$\boldsymbol{u}(t)=\boldsymbol{K}_i(t)\boldsymbol{x}(t)\ .\qquad(2)$$

式中:$\boldsymbol{K}_i(t)=\boldsymbol{K}_i+\Delta\boldsymbol{K}_i(t)$ 为不确定状态反馈增益,$\boldsymbol{K}_i$ 是特定的状态反馈控制器增益,$\Delta\boldsymbol{K}_i\in\mathbf{R}^{m\times n}$ $(i\in S)$ 是控制器增益的摄动.本文考虑加性摄动:

$$\Delta\boldsymbol{K}_i=\boldsymbol{H}_{ki}\boldsymbol{F}_{ki}(t)\boldsymbol{M}_{ki}.$$

式中:$\boldsymbol{H}_{ki},\boldsymbol{M}_{ki}$ 是已知定常矩阵;$\boldsymbol{F}_{ki}(t)$ 是参数不确定矩阵,且满足 $\boldsymbol{F}_{ki}^{\mathrm{T}}(t)\boldsymbol{F}_{ki}(t)\leqslant\boldsymbol{I}$.对于状态反馈控制器(2),有闭环系统:

$$\begin{aligned}\mathrm{d}\boldsymbol{x}(t)=&[(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i(t))\boldsymbol{x}(t)+\boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-\\&\tau(t))]\mathrm{d}t+[\boldsymbol{W}_i\boldsymbol{x}(t)+\boldsymbol{W}_{\mathrm{d}i}\boldsymbol{x}(t-\\&\tau(t))]\mathrm{d}w(t)\ ,\\&\boldsymbol{x}(\tau+\theta)=\boldsymbol{\varphi}(\theta),\theta\in[-\tau,0].\end{aligned}\qquad(3)$$

定义 1 对于任意的初始模态 g_0 和初始状态 $\boldsymbol{\varphi}(\theta)$,存在一个正的标量参数 $T(\boldsymbol{\varphi}(\theta),g_0)$ 使得下式成立:

$$\lim_{T_f\to\infty}E\left\{\int_0^{T_f}\|\boldsymbol{x}(t)\|^2\mathrm{d}t\,|\,\boldsymbol{\varphi}(\theta),g_0\right\}\leqslant T(\boldsymbol{\varphi}(\theta),g_0)\ ,\qquad(4)$$

那么系统(3)是随机稳定的.

定义 2 定义系统(3)的 Lyapunov - Krasovskii 泛函为 $\boldsymbol{V}(\boldsymbol{x}(t),i)$,其无穷小算子为

$$\Gamma\boldsymbol{V}(\boldsymbol{x}(t),i)=\lim_{\Delta t\to0}\frac{1}{\Delta t}[\boldsymbol{E}\{\boldsymbol{V}(\boldsymbol{x}(t+\Delta t),g(t+\Delta t))\,|\,\boldsymbol{x}(t),g(t)=i\}-\boldsymbol{V}(\boldsymbol{x}(t),g(t)=i)]\ .$$

引理 1 给定任意实数 ε 以及方阵 $\boldsymbol{R}$,对于任意矩阵 $\boldsymbol{F}$,有

$$\varepsilon(\boldsymbol{R}+\boldsymbol{R}^{\mathrm{T}})\leqslant\varepsilon^2\boldsymbol{F}+\boldsymbol{R}\boldsymbol{F}^{-1}\boldsymbol{R}^{\mathrm{T}}.$$

引理 2 给定适当维数矩阵 $\boldsymbol{D},\boldsymbol{E}$ 和 $\boldsymbol{F}$,并且 $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I}$,那么,对于任意正标量 ε 有

$$\boldsymbol{MFN}+\boldsymbol{N}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{M}^{\mathrm{T}}\leqslant\varepsilon\boldsymbol{MM}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}.$$

2 弹性状态反馈控制器设计

定理 1 一般不确定转移速率下的闭环随机时滞 Markov 切换系统(3)在状态反馈控制器遭遇加性摄动时是随机稳定的,如果存在对称正定矩阵 $\boldsymbol{P}_i,\boldsymbol{Q}_1,\boldsymbol{Q}_2,\boldsymbol{Q}_{1i},\boldsymbol{Q}_{2i}\in\mathbf{R}^{n\times n}$,对称矩阵 $\boldsymbol{R}_i,\boldsymbol{V}_{1i},\boldsymbol{V}_{2i}\in\mathbf{R}^{n\times n}$,非奇异矩阵 $\boldsymbol{L}_i\in\mathbf{R}^{m\times m}$,$\boldsymbol{F}_i\in\mathbf{R}^{m\times n}$,正常量 ε'_i 使下列不等式成立:

$$P_iB_i = B_iL_i, \tag{5}$$

$$\begin{bmatrix} \boldsymbol{\Pi}_{5i} & P_iA_{di} + W_i^{T}P_iW_{di} & 0 & P_iB_iH_{ki} & \boldsymbol{\Pi}_{6i} \\ * & -(1-h)Q_{1i} + W_{di}^{T}P_iW_{di} & 0 & 0 & 0 \\ * & * & -Q_{2i} & 0 & 0 \\ * & * & * & -\varepsilon_i' I & 0 \\ * & * & * & * & -\boldsymbol{\Pi}_{9i} \end{bmatrix} < 0 ; \tag{6}$$

$$\begin{bmatrix} \sum_{j\in S_k^i}\pi_{ij}(Q_{1j} - V_{1i}) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I - Q_1 & \boldsymbol{\Pi}_{7i} \\ * & -\Pi_{9i} \end{bmatrix} < 0 ; \tag{7}$$

$$\begin{bmatrix} \sum_{j\in S_k^i}\pi_{ij}(Q_{2j} - V_{2i}) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I - Q_2 & \boldsymbol{\Pi}_{8i} \\ * & -\boldsymbol{\Pi}_{9i} \end{bmatrix} < 0 ; \tag{8}$$

$$P_j - R_i \geqslant 0, Q_{1j} - V_{1i} \geqslant 0, Q_{2j} - V_{2i} \geqslant 0, j \in S_{uk}^i, j = i ; \tag{9}$$

$$P_j - R_i \leqslant 0, Q_{1j} - V_{1i} \leqslant 0, Q_{2j} - V_{2i} \leqslant 0, j \in S_{uk}^i, j \neq i . \tag{10}$$

式中：

$\boldsymbol{\Pi}_{5i} = A_i^{T}P_i + F_i^{T}B_i^{T} + B_iF_i + P_iA_i + Q_{1i} + \tau(Q_1 + Q_2) + Q_{2i} + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I + \sum_{j\in S_k^i}\pi_{ij}(P_j - R_i) + W_i^{T}P_iW_i + \varepsilon_i' M_{ki}^{T}M_{ki}$;

$\boldsymbol{\Pi}_{6i} = [P_{k_1^i} - R_i, P_{k_2^i} - R_i, \cdots, P_{k_m^i} - R_i]$;

$\boldsymbol{\Pi}_{7i} = [Q_{1k_1^i} - V_{1i}, Q_{1k_2^i} - V_{1i}, \cdots, Q_{1k_m^i} - V_{1i}]$;

$\boldsymbol{\Pi}_{8i} = [Q_{2k_1^i} - V_{2i}, Q_{2k_2^i} - V_{2i}, \cdots, Q_{2k_m^i} - V_{2i}]$;

$\boldsymbol{\Pi}_{9i} = \mathrm{diag}\{I_{k_1^i}, I_{k_2^i}, \cdots, I_{k_m^i}\}$;

$I_{k_1^i} = I_{k_2^i} = \cdots = I_{k_m^i} = I$.

这里状态反馈控制器增益矩阵为

$$K_i = L_i^{-1}F_i. \tag{11}$$

证明　对系统(3), 构造 Lyapunov – Krasovskii 泛函:

$$V(x(t),i) = x^{T}(t)P_ix(t) + \int_{t-\tau(t)}^{t}x^{T}(s)Q_{1i}x(s)\mathrm{d}s + \int_{t-\tau}^{t}x^{T}(s)Q_{2i}x(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t}x^{T}(s)Q_1x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t}x^{T}(s)Q_2x(s)\mathrm{d}s\mathrm{d}\theta . \tag{12}$$

式中：

$$\sum_{j=1}^{N}\hat{\pi}_{ij}Q_{1j} \leqslant Q_1 ; \sum_{j=1}^{N}\hat{\pi}_{ij}Q_{2j} \leqslant Q_2 . \tag{13}$$

首先,考虑 $F_{ki}(t) = 0$ 时随机时滞 Markov 切换系统(3)的稳定性条件. 由定义 2 得无穷小算子:

$$\begin{aligned} \Gamma V(x(t),i) = {} & x^{T}(t)\Big[(A_i + B_iK_i)^{T}P_i + P_i(A_i + B_iK_i) + \sum_{j=1}^{N}\hat{\pi}_{ij}P_j\Big]x(t) + x^{T}(t)P_iA_{di}x(t-\tau(t)) + \\ & x^{T}(t-\tau(t))A_{di}^{T}P_ix(t) + x^{T}(t)Q_{1i}x(t) + \\ & [W_ix(t) + W_{di}x(t-\tau(t))]^{T}P_i[W_ix(t) + W_{di}x(t-\tau(t))] - (1-\dot{\tau}(t))x^{T}(t-\tau(t))Q_{1i}x(t-\tau(t)) + \sum_{j=1}^{N}\hat{\pi}_{ij}\int_{t-\tau(t)}^{t}x^{T}(s)Q_{1j}x(s)\mathrm{d}s + \\ & \tau x^{T}(t)Q_1x(t) - \int_{t-\tau}^{t}x^{T}(s)Q_1x(s)\mathrm{d}s + x^{T}(t)Q_{2i}x(t) - x^{T}(t-\tau)Q_{2i}x(t-\tau) + \tau x^{T}(t)Q_2x(t) + \\ & \sum_{j=1}^{N}\hat{\pi}_{ij}\int_{t-\tau}^{t}x^{T}(s)Q_{2j}x(s)\mathrm{d}s - \int_{t-\tau}^{t}x^{T}(s)Q_2x(s)\mathrm{d}s . \end{aligned}$$

由于 $\sum_{j=1}^{N}\hat{\pi}_{ij}R_i = \sum_{j=1}^{N}\hat{\pi}_{ij}V_{1i} = \sum_{j=1}^{N}\hat{\pi}_{ij}V_{2i} = 0$, 因此,

$$\begin{aligned} \sum_{j=1}^{N}\hat{\pi}_{ij}P_j = {} & \sum_{j=1}^{N}\hat{\pi}_{ij}(P_j - R_i) = \\ & \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\pi_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\Delta\pi_{ij}(P_j - R_i) = \\ & \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\pi_{ij}(P_j - R_i) + \\ & \sum_{j\in S_k^i}\frac{1}{2}\Delta\pi_{ij}[(P_j - R_i) + (P_j - R_i)^{T}] . \end{aligned}$$

根据引理 1 可得

$$\sum_{j=1}^{N}\hat{\pi}_{ij}(P_j - R_i) \leqslant \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\pi_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I + (P_j - R_i)(P_j - R_i)^{T}. \tag{14}$$

同理可得

$$\sum_{j=1}^{N}\hat{\pi}_{ij}Q_{1j} - Q_1 \leqslant \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(Q_{1j} - V_{1i}) + \sum_{j\in S_k^i}\pi_{ij}(Q_{1j} - V_{1i}) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I + (Q_{1j} - V_{1i})(Q_{1j} - V_{1i})^{T} \leqslant 0 , \tag{15}$$

$$\sum_{j=1}^{N}\hat{\pi}_{ij}Q_{2j} - Q_2 \leqslant \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(Q_{2j} - V_{2i}) + \sum_{j\in S_k^i}\pi_{ij}(Q_{2j} - V_{2i}) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2 I + (Q_{2j} - V_{2i})(Q_{2j} - V_{2i})^{T} \leqslant 0 . \tag{16}$$

结合式(12),式(13)及条件 $\dot{\tau}(t) \leqslant \tau$ 可得

$$\Gamma V(x(t),i) < x^{T}(t)\Big[(A_i + B_iK_i)^{T}P_i + P_i(A_i + B_iK_i) + \sum_{j\in S_{uk}^i}\hat{\pi}_{ij}(P_j - R_i) + \sum_{j\in S_k^i}\pi_{ij}(P_j -$$

$\boldsymbol{R}_i) + \sum_{j\in S_k^i}\frac{1}{4}\lambda_{ij}^2\boldsymbol{I} + (\boldsymbol{P}_j - \boldsymbol{R}_i)(\boldsymbol{P}_j - \boldsymbol{R}_i)^{\mathrm{T}}]\boldsymbol{x}(t) +$
$\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_i\boldsymbol{A}_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t))+\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{A}_{\mathrm{d}i}^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{x}(t) -$
$(1-h)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_{1i}\boldsymbol{x}(t-\tau(t)) - \boldsymbol{x}^{\mathrm{T}}(t-$
$\tau)\boldsymbol{Q}_{2i}\boldsymbol{x}(t-\tau) + [\boldsymbol{W}_i\boldsymbol{x}(t) + \boldsymbol{W}_{\mathrm{d}i}\boldsymbol{x}(t-$
$\tau(t))]^{\mathrm{T}}\boldsymbol{P}_i[\boldsymbol{W}_i\boldsymbol{x}(t) + \boldsymbol{W}_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t))] +$
$\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_{1i}\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_{2i}\boldsymbol{x}(t) +$
$\tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t) + \tau\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t) = \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Pi}_{1i}\boldsymbol{\Psi}.$

式中:

$$\boldsymbol{\Psi} = [\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \quad \boldsymbol{x}^{\mathrm{T}}(t-\tau)]^{\mathrm{T}};$$

$$\boldsymbol{\Pi}_{1i} = \begin{bmatrix} \boldsymbol{\Pi}_{2i} & \boldsymbol{P}_i\boldsymbol{A}_{\mathrm{d}i} + \boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_{\mathbf{i}}\boldsymbol{W}_{\mathrm{d}i} & 0 \\ * & -(1-h)\boldsymbol{Q}_{1i} + \boldsymbol{W}_{\mathrm{d}i}^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_{\mathrm{d}i} & 0 \\ * & * & -\boldsymbol{Q}_{2i} \end{bmatrix};$$

$$\boldsymbol{\Pi}_{2i} = (\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i(\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i) + \sum_{j\in S_{\mathrm{uk}}^i}\pi_{ij}(\boldsymbol{P}_j - \boldsymbol{R}_i) + \sum_{j\in S_k^i}\pi_{ij}(\boldsymbol{P}_j - \boldsymbol{R}_i) + \sum_{j\in S_k^i}[\frac{1}{4}\lambda_{ij}^2\boldsymbol{I} + (\boldsymbol{P}_j - \boldsymbol{R}_i)(\boldsymbol{P}_j - \boldsymbol{R}_i)^{\mathrm{T}}] + \boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_i + \boldsymbol{Q}_{1i} + \tau(\boldsymbol{Q}_1 + \boldsymbol{Q}_2) + \boldsymbol{Q}_{2i}.$$

显然,当 $\boldsymbol{\Pi}_{1i}<0$ 时,$\Gamma\boldsymbol{V}(\boldsymbol{x}(t),i)<0$. 所以,$\Gamma\boldsymbol{V}(\boldsymbol{x}(t),i) \leqslant -\min_{i\in S}\{\lambda_{\min}(-\boldsymbol{\Pi}_2)\}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)$.

根据定义 1 可知,闭环系统(3)是随机稳定的.

其次,考虑 $\boldsymbol{F}_{ki}(t)\neq 0$ 的情况,即设计弹性状态反馈控制器(2)保证闭环系统(3)的随机稳定性.

令 $\boldsymbol{\Pi}_{1i}$ 中 $\boldsymbol{K}_i = \boldsymbol{K}_i + \Delta\boldsymbol{K}_i(t)$,其中,$\Delta\boldsymbol{K}_i = \boldsymbol{H}_{ki}\boldsymbol{F}_{ki}(t)\boldsymbol{M}_{ki}$,可得

$$\Gamma\boldsymbol{V}(\boldsymbol{x}(t),i) \leqslant \boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Pi}_{3i}\boldsymbol{\Psi}. \tag{17}$$

式中:

$$\boldsymbol{\Pi}_{3i} = \begin{bmatrix} \boldsymbol{\Pi}_{4i} & \boldsymbol{P}_i\boldsymbol{A}_{\mathrm{d}i} + \boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_{\mathrm{d}i} & 0 \\ * & -(1-h)\boldsymbol{Q}_{1i} + \boldsymbol{W}_{\mathrm{d}i}^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{W}_{\mathrm{d}i} & 0 \\ * & * & -\boldsymbol{Q}_{2i} \end{bmatrix};$$

$$\boldsymbol{\Pi}_{4i} = \boldsymbol{\Pi}_{2i} + (\boldsymbol{B}_i\boldsymbol{H}_{ki}\boldsymbol{F}_{ki}(t)\boldsymbol{M}_{ki})^{\mathrm{T}}\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{H}_{ki}\times\boldsymbol{F}_{ki}(t)\boldsymbol{M}_{ki} \leqslant \boldsymbol{\Pi}_{2i} + \varepsilon_i\boldsymbol{P}_i\boldsymbol{B}_i\boldsymbol{H}_{ki}\boldsymbol{H}_{ki}^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}_i + \varepsilon_i^{-1}\boldsymbol{M}_{ki}^{\mathrm{T}}\boldsymbol{M}_{ki}.$$

令 $\boldsymbol{K}_i = \boldsymbol{L}_i^{-1}\boldsymbol{F}_i$,$\boldsymbol{P}_i\boldsymbol{B}_i = \boldsymbol{B}_i\boldsymbol{L}_i$,$\varepsilon_i' = \varepsilon_i$,代入 $\boldsymbol{\Pi}_{3i}$ 并利用 Schur 补引理可知,当条件(5)~(10)被满足时,根据定义 1 可知,一般不确定转移速率的闭环随机时滞 Markov 切换系统(3)是随机稳定的. 定理 1 证明完毕.

注 1 当 $\Delta\pi_{ij}=0$ 时,闭环系统(3)退化为转移速率部分未知的情况.

注 2 将式(2)中的 $\boldsymbol{K}_i$ 替换成 $\boldsymbol{K}$,问题就转化为求解模态独立控制器.

注 3 式(5)中变量 $\boldsymbol{L}_i$ 需要满足的 LMI 求解条件:

$$[\boldsymbol{P}_i\boldsymbol{B}_i - \boldsymbol{B}_i\boldsymbol{L}_i]^{\mathrm{T}}[\boldsymbol{P}_i\boldsymbol{B}_i - \boldsymbol{B}_i\boldsymbol{L}_i] \leqslant \delta_i\boldsymbol{I}.$$

式中 δ_i 是足够小的正常量. 那么相应 LMI 为

$$\begin{bmatrix} -\delta_i\boldsymbol{I} & [\boldsymbol{P}_i\boldsymbol{B}_i - \boldsymbol{B}_i\boldsymbol{L}_i]^{\mathrm{T}} \\ * & -\boldsymbol{I} \end{bmatrix} \leqslant 0.$$

3 仿真算例

考虑如下四模态的随机 Markov 切换系统,系统参数如下:

$$\boldsymbol{A}_1 = \begin{bmatrix} -2.2 & 0.6 \\ 0.9 & -2.0 \end{bmatrix}, \boldsymbol{A}_2 = \begin{bmatrix} -2.5 & 1.2 \\ 1.5 & -1.8 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0.6 & 1.2 \\ 0.5 & -1.5 \end{bmatrix}, \boldsymbol{A}_4 = \begin{bmatrix} -2.0 & 1.2 \\ 0.8 & -1.5 \end{bmatrix},$$

$$\boldsymbol{A}_{\mathrm{d1}} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \boldsymbol{A}_{\mathrm{d2}} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$\boldsymbol{A}_{\mathrm{d3}} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \boldsymbol{A}_{\mathrm{d1}} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}_3 = \boldsymbol{B}_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$\boldsymbol{W}_{\mathrm{d1}} = \boldsymbol{W}_{\mathrm{d2}} = \boldsymbol{W}_{\mathrm{d3}} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\boldsymbol{W}_{\mathrm{d4}} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix},$$

$$\boldsymbol{H}_{k1} = \boldsymbol{H}_{k2} = \boldsymbol{H}_{k3} = \boldsymbol{H}_{k4} = [0.5 \quad 0],$$

$$\boldsymbol{M}_{k1} = \boldsymbol{M}_{k2} = \boldsymbol{M}_{k3} = \boldsymbol{M}_{k4} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$\tau = 1.0$,$h = 0.5$,$\boldsymbol{x}(0) = [-0.5 \quad 1.2]^{\mathrm{T}}$,$\boldsymbol{x}(t) = [0 \quad 0]^{\mathrm{T}}$,$t\in[-\tau,0)$,$g_0 = 4$.

系统的一般不确定转移速率矩阵为

$$\begin{bmatrix} -1.8+\Delta\pi_{11} & ? & ? & 0.8+\Delta\pi_{14} \\ ? & ? & 0.3+\Delta\pi_{23} & 0.4+\Delta\pi_{24} \\ 0.2+\Delta\pi_{31} & ? & -2.0+\Delta\pi_{33} & ? \\ 0.2+\Delta\pi_{41} & ? & 0.5+\Delta\pi_{43} & ? \end{bmatrix}.$$

令 $\Delta\pi_{ij} \leqslant \lambda_{ij} = |0.2\times\pi_{ij}|$,通过求解定理 1,可得弹性控制器增益参数:

$$\boldsymbol{K}_1 = [-72.2796 \quad -89.0475],$$
$$\boldsymbol{K}_2 = [-78.2064 \quad -123.3440],$$
$$\boldsymbol{K}_3 = [-173.9671 \quad -51.5053],$$
$$\boldsymbol{K}_4 = [-193.3942 \quad -204.7072].$$

将此控制器应用于原系统,可得系统的状态轨迹如图 1 所示.

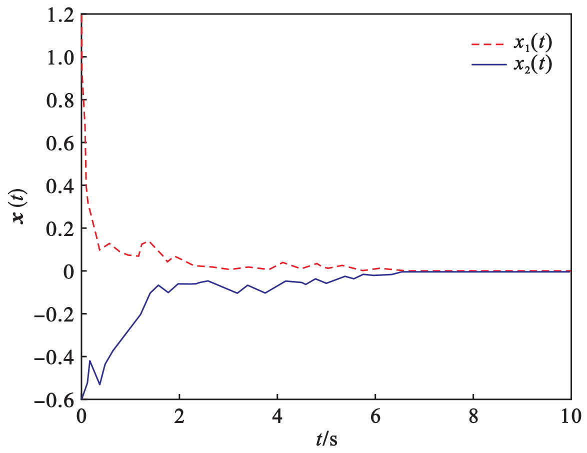

图 1　状态轨迹
Fig. 1　State trajectories

仿真结果表明，在所得弹性状态反馈控制器的作用下，算例给出的闭环系统状态 $\boldsymbol{x}(t)$ 尽管在最初时刻表现为震荡，但是在 7 s 之内可以迅速收敛，达到稳定.

4　结　　论

本文针对具有一般不确定转移速率的随机时滞 Markov 切换系统，研究了模态依赖型的弹性控制器设计问题. 首先，构建了适当的 Lyapunov – Krasovskii 泛函，在线性矩阵不等式的框架下，实现了弹性控制器的求解与证明. 最后，利用数值仿真验证了所得结果的有效性. 本文所研究的系统较转移速率不完全已知的情况更具有一般性，并且控制器和 Lyapunov – Krasovskii 均是模态依赖型，所得结果相对模态独立型具有较低的保守性.

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