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不确定多时滞切换广义系统的鲁棒无源控制

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摘　　要: 将无源的概念从广义系统扩展到带有多时滞的切换广义系统之中,进而研究了一类同时具有不确定项和多时滞项的相对比较复杂的系统的无源控制问题,其中一些条件需满足假设前提. 首先,利用一种广义 Lyapunov 方法再结合线性矩阵不等式方法,给出了使不确定多时滞切换广义系统能够渐近稳定且严格无源的充分条件; 再根据已有的条件设计出鲁棒无源控制器,使得带有不确定项和多时滞项的切换广义系统可以渐近稳定并严格无源. 最后用数值算例说明了有效性.

关 键 词: 多时滞;鲁棒无源控制;切换广义系统;不确定性

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Robust Passive Control for Uncertain Switched Singular Systems with Multiple Time-Delays

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Abstract: The concept of passive source is extended to the switched singular systems with multiple time-delays. The problem of robust passive control for a class of switched singular systems with both uncertainties and multiple time-delays is studied, and some of these conditions need to satisfy the hypothesis. First, by means of generalized Lyapunov function and linear matrix inequality, the sufficient conditions are given for the asymptotic stability and strictly passive of uncertain switched singular systems with multiple time-delays. Moreover, the qualified robust passive controller is designed according to the existing conditions, so that the switched singular systems can be asymptotically stable and strictly passive. Finally, numerical examples illustrate the effectiveness of the approaches.

Key words: multiple time-delays; robust passive control; switched singular systems; uncertain

近些年,关于广义系统的无源控制问题的研究很多,而对切换广义系统无源控制问题的研究还鲜有成果[1-7]. 本文研究了带有不确定和多时滞的相对比较复杂的系统的无源控制问题. 利用广义 Lyapunov 函数方法再结合线性矩阵不等式方法,得到了使切换广义系统渐近稳定且严格无源的充分条件,同时设计出了鲁棒无源控制器.

1　问题描述与预备知识

$$\left.\begin{aligned}&\boldsymbol{E}\dot{\boldsymbol{x}}(t)=[\boldsymbol{A}_{\sigma(t)}+\Delta\boldsymbol{A}_{\sigma(t)}]\boldsymbol{x}(t)+[\boldsymbol{A}_{1\sigma(t)}+\Delta\boldsymbol{A}_{1\sigma(t)}]\boldsymbol{x}(t-d_1)+\\&[\boldsymbol{A}_{2\sigma(t)}+\Delta\boldsymbol{A}_{2\sigma(t)}]\boldsymbol{x}(t-d_2)+\boldsymbol{B}_{\sigma(t)}\boldsymbol{u}(t)+[\boldsymbol{G}_{1\sigma(t)}+\Delta\boldsymbol{G}_{1\sigma(t)}]\boldsymbol{\omega}(t)\ ,\\&\boldsymbol{z}(t)=\boldsymbol{C}_{\sigma(t)}\boldsymbol{x}(t)+\boldsymbol{D}_{\sigma(t)}\boldsymbol{u}(t)+\boldsymbol{G}_{2\sigma(t)}\boldsymbol{\omega}(t)\ ,\\&\boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\xi,0]\ .\end{aligned}\right\}\tag{1}$$

式中:$\boldsymbol{x}(t)$为状态向量;$\boldsymbol{u}(t)$为控制输入向量;$\boldsymbol{z}(t)$为控制输出向量;$\boldsymbol{\omega}(t)\in L_2[0,+\infty]$,$\boldsymbol{\omega}(t)$为外部扰动向量;$\boldsymbol{\varphi}(t)$表示$[-\xi,0]$上的连续初始状态;$d$ 表示正时滞常数;$\sigma(\cdot):[0,+\infty]\rightarrow$

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$\{1,2\cdots,n\}\overset{\text{def}}{=}\bar{N}$，表示分段常值切换信号，且 $\sigma(t)=i$ 表示第 i 个子系统在 t 时刻被激活；$\boldsymbol{E}$，$\boldsymbol{A}_i,\boldsymbol{A}_{1i},\boldsymbol{A}_{2i},\boldsymbol{B}_i\boldsymbol{C}_i,\boldsymbol{D}_i,\boldsymbol{G}_{1i},\boldsymbol{G}_{2i}$ 分别表示适当维数的时常矩阵. 其中 $\Delta\boldsymbol{A}_i,\Delta\boldsymbol{A}_{1i},\Delta\boldsymbol{A}_{2i},\Delta\boldsymbol{G}_{1i}$ 表示参数不确定矩阵，且满足条件：

$$[\Delta\boldsymbol{A}_i,\Delta\boldsymbol{A}_{1i},\Delta\boldsymbol{A}_{2i},\Delta\boldsymbol{G}_{1i}]=\boldsymbol{N}_i\boldsymbol{F}_i(t)[\boldsymbol{H}_i,\boldsymbol{H}_{1i},\boldsymbol{H}_{2i},\boldsymbol{T}_{1i}],\ \forall i\in\mathbf{N}. \tag{2}$$

其中 $\boldsymbol{N}_i,\boldsymbol{H}_i,\boldsymbol{H}_{1i},\boldsymbol{H}_{2i},\boldsymbol{T}_{1i}$ 分别表示相应维数的时常矩阵；$\boldsymbol{F}_i(t)$ 表示不确定矩阵，且满足：

$$\boldsymbol{F}_i^{\mathrm{T}}(t)\boldsymbol{F}_i(t)\leqslant\boldsymbol{I}. \tag{3}$$

本文所研究的切换广义系统为正则且无脉冲的.

定义 1 切换广义系统(1)是鲁棒无源的，如果存在一个非负函数 $V(x)\geqslant 0$，使得无源不等式 $\dot{V}(x)\leqslant\boldsymbol{\omega}^{\mathrm{T}}(t)z(t)$，$\forall t\geqslant 0$ 对任意扰动输入 $\boldsymbol{\omega}(t)$ 都能成立，且满足式(2)、式(3). 假若无源不等式换为严格不等式，那么称切换广义系统是严格无源的.

引理 1[8] (Schur 补引理) 对于给出的对称矩阵 $\boldsymbol{S}=\begin{bmatrix}\boldsymbol{S}_{11} & \boldsymbol{S}_{12}\\ \boldsymbol{S}_{12}^{\mathrm{T}} & \boldsymbol{S}_{22}\end{bmatrix}$，$\boldsymbol{S}_{11}$ 是 $r\times r$ 维的，那么下面 3 个不等式一定是等价的：

① $\boldsymbol{S}<\boldsymbol{0}$，

② $\boldsymbol{S}_{11}<\boldsymbol{0}$，$\boldsymbol{S}_{22}-\boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}<\boldsymbol{0}$，

③ $\boldsymbol{S}_{22}<\boldsymbol{0}$，$\boldsymbol{S}_{11}-\boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}}<\boldsymbol{0}$.

引理 2[9] 给定适当维数的矩阵 $\boldsymbol{X},\boldsymbol{N},\boldsymbol{H}$，若有 $\boldsymbol{X}+\boldsymbol{NFH}+\boldsymbol{H}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}<\boldsymbol{0}$，对于所有满足 $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leqslant\boldsymbol{I}$ 都成立，则存在 $\varepsilon>0$，定有下式成立：

$$\boldsymbol{X}+\varepsilon\boldsymbol{NN}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}<\boldsymbol{0}.$$

2 无源性分析

系统在状态反馈控制器 $\boldsymbol{u}_i=\boldsymbol{K}_i\boldsymbol{x}(t)$ 作用下的闭环系统为

$$\left.\begin{aligned}&\boldsymbol{E}\dot{\boldsymbol{x}}(t)=[\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i]\boldsymbol{x}(t)+\hat{\boldsymbol{A}}_{1i}\boldsymbol{x}(t-d_1)+\\&\qquad\hat{\boldsymbol{A}}_{2i}\boldsymbol{x}(t-d_2)+\hat{\boldsymbol{G}}_{1i}\boldsymbol{\omega}(t),\\&z(t)=[\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i]\boldsymbol{x}(t)+\boldsymbol{G}_{2i}\boldsymbol{\omega}(t),\\&\boldsymbol{x}(t)=\boldsymbol{\varphi}(t),t\in[-\xi,0].\end{aligned}\right\} \tag{4}$$

令 $\hat{\boldsymbol{A}}_i=\boldsymbol{A}_i+\Delta\boldsymbol{A}_i,\hat{\boldsymbol{A}}_{1i}=\boldsymbol{A}_{1i}+\Delta\boldsymbol{A}_{1i},\hat{\boldsymbol{A}}_{2i}=\boldsymbol{A}_{2i}+\Delta\boldsymbol{A}_{2i}$，$\hat{\boldsymbol{G}}_{1i}=\boldsymbol{G}_{1i}+\Delta\boldsymbol{G}_{1i}$，且 * 表示关于对角线的对称矩阵部分.

定理 1 如果 $\boldsymbol{G}_{2i}+\boldsymbol{G}_{2i}^{\mathrm{T}}>\boldsymbol{0}$，并且存在可逆矩阵 $\boldsymbol{P}\in\mathbf{R}^{n\times n}$，同时也存在适当维数矩阵 $\boldsymbol{K}_i$ 以及对称正定矩阵 $\boldsymbol{Q}_1,\boldsymbol{Q}_2$，使不等式成立：

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}\boldsymbol{E}\geqslant\boldsymbol{0}, \tag{5}$$

$$\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{G}}_{1i}^{\mathrm{T}}-(\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}}\end{bmatrix}<\boldsymbol{0}. \tag{6}$$

$\boldsymbol{\Gamma}=(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$.

此时切换广义系统(1)在任意切换律下对任意扰动是渐近稳定且严格无源的.

证明 构造广义 Lyapunov 函数

$$\boldsymbol{V}_1(\boldsymbol{x})=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\int_{t-d_1}^{t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{Q}_1\boldsymbol{x}(\tau)\mathrm{d}\tau+\int_{t-d_2}^{t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{Q}_2\boldsymbol{x}(\tau)\mathrm{d}\tau. \tag{7}$$

很明显 $\boldsymbol{V}_1(\boldsymbol{x})$ 是正定的，再对系统沿轨迹对 t 求导，可得

$$\begin{aligned}&\dot{\boldsymbol{V}}_1(\boldsymbol{x})=[\boldsymbol{E}\dot{\boldsymbol{x}}(t)]^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}[\boldsymbol{E}\dot{\boldsymbol{x}}(t)]+\\&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_1)\boldsymbol{Q}_1\boldsymbol{x}(t-d_1)+\\&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_2)\boldsymbol{Q}_2\boldsymbol{x}(t-d_2)=\\&\boldsymbol{x}^{\mathrm{T}}(t)(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t-d_1)\hat{\boldsymbol{A}}_{1i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\\&\boldsymbol{x}^{\mathrm{T}}(t-d_2)\hat{\boldsymbol{A}}_{2i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{x}(t)+\\&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i}\boldsymbol{x}(t-d_1)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i}\boldsymbol{x}(t-d_2)+\\&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_1)\boldsymbol{Q}_1\boldsymbol{x}(t-d_1)+\\&\boldsymbol{\omega}^{\mathrm{T}}(t)\hat{\boldsymbol{G}}_{1i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{G}}_{1i}\boldsymbol{\omega}(t)+\\&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_2)\boldsymbol{Q}_2\boldsymbol{x}(t-d_2)=\\&\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\\ \boldsymbol{\omega}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}_{1i}^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0}\\ * & * & * & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\\ \boldsymbol{\omega}(t)\end{bmatrix}.\end{aligned}$$

其中：

$\boldsymbol{\Gamma}=(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$.

首先讨论系统的渐近稳定性.

当不考虑外部扰动，即当 $\boldsymbol{\omega}(t)=0$ 时，

$$\dot{\boldsymbol{V}}_1(\boldsymbol{x})=\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\end{bmatrix},$$

此时式(6)退化为 $\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2\end{bmatrix}<\boldsymbol{0}$，

故可得 $\dot{\boldsymbol{V}}_1(\boldsymbol{x})<\boldsymbol{0}$. 根据 Lyapunov 稳定性理论，可知系统(1)是渐近稳定的.

下面考虑系统无源性.

$$\begin{aligned}&\dot{\boldsymbol{V}}_1(\boldsymbol{x})-\boldsymbol{\omega}^{\mathrm{T}}(t)z(t)=\\&\dot{\boldsymbol{V}}_1(\boldsymbol{x})-\boldsymbol{\omega}^{\mathrm{T}}(t)z(t)-z^{\mathrm{T}}(t)\boldsymbol{\omega}(t)+z^{\mathrm{T}}(t)\boldsymbol{\omega}(t)=\\&\boldsymbol{x}^{\mathrm{T}}(t)(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t-d_1)\hat{\boldsymbol{A}}_{1i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\end{aligned}$$

$$
\begin{aligned}
&\boldsymbol{x}^{\mathrm{T}}(t-d_2)\hat{\boldsymbol{A}}_{2i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)\boldsymbol{x}(t)+\\
&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i}\boldsymbol{x}(t-d_1)+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i}\boldsymbol{x}(t-d_2)+\\
&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_1)\boldsymbol{Q}_1\boldsymbol{x}(t-d_1)+\\
&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_2\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t)[\boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{G}}_{1i}-\\
&(\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}}]\boldsymbol{\omega}(t)-\boldsymbol{x}^{\mathrm{T}}(t-d_2)\boldsymbol{Q}_2\boldsymbol{x}(t-d_2)+\\
&\boldsymbol{\omega}^{\mathrm{T}}(t)[\hat{\boldsymbol{G}}_{1i}^{\mathrm{T}}\boldsymbol{P}-(\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i)]\boldsymbol{x}(t)=\\
&\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\\ \boldsymbol{\omega}(t)\end{bmatrix}^{\mathrm{T}}\boldsymbol{\Theta}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-d_1)\\ \boldsymbol{x}(t-d_2)\\ \boldsymbol{\omega}(t)\end{bmatrix}+z^{\mathrm{T}}(t)\boldsymbol{\omega}(t)\ . \qquad (8)
\end{aligned}
$$

$$
\boldsymbol{\Theta}=\begin{bmatrix}\boldsymbol{\Gamma} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{G}}_{1i}^{\mathrm{T}}-(\hat{\boldsymbol{C}}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}}\end{bmatrix}. \qquad (9)
$$

其中 $\boldsymbol{\Gamma}=(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\hat{\boldsymbol{A}}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$. 当 $\boldsymbol{\Theta}<\boldsymbol{0}$ 成立时，即有

$$
\dot{V}_1(\boldsymbol{x})-2\boldsymbol{\omega}^{\mathrm{T}}(t)z(t)<\boldsymbol{0}\ . \qquad (10)
$$

取 $\boldsymbol{V}(\boldsymbol{x})=\dfrac{1}{2}\boldsymbol{V}_1(\boldsymbol{x})$，此时系统满足定义 1，所以切换广义系统(1)是渐近稳定且严格无源的.

定理 2　如果 $\boldsymbol{G}_{2i}+\boldsymbol{G}_{2i}^{\mathrm{T}}>\boldsymbol{0}$，且存在可逆矩阵 $\boldsymbol{P}\in\mathbf{R}^{n\times n}$，及适当维数矩阵 $\boldsymbol{K}_i$ 和对称正定矩阵 $\boldsymbol{Q}_1,\boldsymbol{Q}_2$，使得如下不等式成立：

$$
\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}\boldsymbol{E}\geqslant\boldsymbol{0}\ . \qquad (11)
$$

$$
\begin{bmatrix}\boldsymbol{\Pi} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{1i} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{2i} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}_{1i}-(\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}} & \varepsilon\boldsymbol{P}^{\mathrm{T}}\boldsymbol{N}_i & \boldsymbol{H}_i^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{1i}^{\mathrm{T}}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{2i}^{\mathrm{T}}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{T}_{1i}^{\mathrm{T}}\\ * & * & * & * & -\varepsilon\boldsymbol{I} & \boldsymbol{0}\\ * & * & * & * & * & -\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0}. \qquad (12)
$$

其中：

$\boldsymbol{\Pi}=(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$.

此时切换广义系统(1)在任意切换律下对任意扰动是渐近稳定且严格无源的.

证明　下面证式(12)与式(6)等价. 将式(2)、式(3)代入式(9)中，得到

$$
\boldsymbol{\Theta}=\boldsymbol{\Phi}+\begin{bmatrix}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{N}_i\\ \boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{F}_i[\boldsymbol{H}_i\quad \boldsymbol{H}_{1i}\quad \boldsymbol{H}_{2i}\quad \boldsymbol{T}_{1i}]+\begin{bmatrix}\boldsymbol{H}_i^{\mathrm{T}}\\ \boldsymbol{H}_{1i}^{\mathrm{T}}\\ \boldsymbol{H}_{2i}^{\mathrm{T}}\\ \boldsymbol{T}_{1i}^{\mathrm{T}}\end{bmatrix}\boldsymbol{F}_i^{\mathrm{T}}[\boldsymbol{N}_i^{\mathrm{T}}\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}]\ . \qquad (13)
$$

其中

$$
\boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\Pi} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{1i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{A}}_{2i} & \boldsymbol{P}^{\mathrm{T}}\hat{\boldsymbol{G}}_{1i}^{\mathrm{T}}-(\hat{\boldsymbol{C}}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}}\end{bmatrix}. \qquad (14)
$$

$\boldsymbol{\Pi}=(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$，故有 $\boldsymbol{\Theta}=\boldsymbol{\Phi}+\boldsymbol{N}\boldsymbol{F}_i\boldsymbol{H}+\boldsymbol{H}^{\mathrm{T}}\boldsymbol{F}_i^{\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}$ 成立.

其中　$\boldsymbol{N}=[\boldsymbol{N}_i^{\mathrm{T}}\boldsymbol{P}\quad \boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{0}]^{\mathrm{T}}$，$\boldsymbol{H}=[\boldsymbol{H}_i\quad \boldsymbol{H}_{1i}\quad \boldsymbol{H}_{2i}\quad \boldsymbol{T}_{1i}]$. 由引理 2 得

$$
\boldsymbol{\Theta}+\varepsilon\boldsymbol{N}\boldsymbol{N}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}<\boldsymbol{0}\ . \qquad (15)
$$

再由引理 1 得到：

$$
\begin{bmatrix}\boldsymbol{\Pi} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{1i} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{2i} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{G}_{1i}-(\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_i)^{\mathrm{T}} & \varepsilon\boldsymbol{P}^{\mathrm{T}}\boldsymbol{N}_i & \boldsymbol{H}_i^{\mathrm{T}}\\ * & -\boldsymbol{Q}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{1i}^{\mathrm{T}}\\ * & * & -\boldsymbol{Q}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{2i}^{\mathrm{T}}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{T}_{1i}^{\mathrm{T}}\\ * & * & * & * & -\varepsilon\boldsymbol{I} & \boldsymbol{0}\\ * & * & * & * & * & -\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0}\ . \qquad (16)
$$

其中

$\boldsymbol{\Pi}=(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i)+\boldsymbol{Q}_1+\boldsymbol{Q}_2$.

故此时切换广义系统(1)在任意切换律下对任意扰动是渐近稳定且严格无源的.

3　无源控制器设计

定理 3　如果 $\boldsymbol{G}_{2i}+\boldsymbol{G}_{2i}^{\mathrm{T}}>\boldsymbol{0}$，并且可以存在 $\varepsilon>0$，同时存在适当维数矩阵 $\boldsymbol{W}_i$，和可逆矩阵 $\boldsymbol{X}$，以及对称且正定矩阵 $\boldsymbol{Q}_{11},\boldsymbol{Q}_{12}$，使不等式成立：

$$
\boldsymbol{X}^{\mathrm{T}}\boldsymbol{E}^{\mathrm{T}}=\boldsymbol{E}\boldsymbol{X}\geqslant\boldsymbol{0}\ , \qquad (17)
$$

$$
\begin{bmatrix}\boldsymbol{\Lambda} & \boldsymbol{A}_{1i}\boldsymbol{X} & \boldsymbol{A}_{2i}\boldsymbol{X} & \boldsymbol{G}_{1i}-(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{C}_i^{\mathrm{T}}+\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}}) & \varepsilon\boldsymbol{N}_i & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{H}_i^{\mathrm{T}}\\ * & -\boldsymbol{Q}_{11} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{H}_{1i}^{\mathrm{T}}\\ * & * & -\boldsymbol{Q}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{X}^{\mathrm{T}}\boldsymbol{H}_{2i}^{\mathrm{T}}\\ * & * & * & -\boldsymbol{G}_{2i}-\boldsymbol{G}_{2i}^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{T}_{1i}^{\mathrm{T}}\\ * & * & * & * & -\varepsilon\boldsymbol{I} & \boldsymbol{0}\\ * & * & * & * & * & -\varepsilon\boldsymbol{I}\end{bmatrix}<\boldsymbol{0}, \qquad (18)
$$

$$\boldsymbol{\Lambda}=\boldsymbol{A}_i\boldsymbol{X}+\boldsymbol{X}^{\mathrm{T}}\boldsymbol{A}_i^{\mathrm{T}}+\boldsymbol{B}_i\boldsymbol{W}_i+\boldsymbol{W}_i^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}}+\boldsymbol{Q}_{11}+\boldsymbol{Q}_{12},$$

那么,此时切换广义系统(1)在任意切换律下对任意扰动是渐近稳定且严格无源的.

证明　将式(12)进行变换,先将其左乘 $\mathrm{diag}\{\boldsymbol{P}^{-\mathrm{T}},\boldsymbol{P}^{-\mathrm{T}},\boldsymbol{P}^{-\mathrm{T}},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$,再右乘 $\mathrm{diag}\{\boldsymbol{P}^{-1},\boldsymbol{P}^{-1},\boldsymbol{P}^{-1},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$,并令 $\boldsymbol{P}^{-1}=\boldsymbol{X}$, $\boldsymbol{K}_i=\boldsymbol{W}_i\boldsymbol{X}^{-1}$, $\boldsymbol{Q}_{11}=\boldsymbol{P}^{-\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{P}^{-1}$, $\boldsymbol{Q}_{12}=\boldsymbol{P}^{-\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{P}^{-1}$,便得到了式(18).故得证.

4 数值算例

$$\boldsymbol{E}=\begin{bmatrix}1&0\\0&0\end{bmatrix},\boldsymbol{A}_1=\begin{bmatrix}-0.3&0\\0&0.3\end{bmatrix},$$

$$\boldsymbol{A}_2=\begin{bmatrix}-1.1&0\\0&1.5\end{bmatrix},$$

$$\boldsymbol{B}_1=\begin{bmatrix}0.2\\1.4\end{bmatrix},\boldsymbol{B}_2=\begin{bmatrix}0.2\\0.3\end{bmatrix},$$

$$\boldsymbol{A}_{11}=\boldsymbol{A}_{21}=\begin{bmatrix}0&1.2\\1.3&2\end{bmatrix},$$

$$\boldsymbol{A}_{12}=\boldsymbol{A}_{22}=\begin{bmatrix}0.2&0.9\\1.1&1.5\end{bmatrix},\boldsymbol{C}_1=[0.8\quad 0.9],$$

$$\boldsymbol{C}_2=[0.7\quad 0.8],\boldsymbol{D}_1=\boldsymbol{D}_2=0.4,\lambda=0.5$$

$$\boldsymbol{H}_1=\begin{bmatrix}0.2&0\\0.3&1\end{bmatrix},\boldsymbol{H}_{11}=\begin{bmatrix}-0.1&1\\0.5&3\end{bmatrix},$$

$$\boldsymbol{H}_{21}=\begin{bmatrix}-0.3&0\\0.6&4\end{bmatrix},$$

$$\boldsymbol{H}_2=\begin{bmatrix}0.1&0\\0.4&1\end{bmatrix},\boldsymbol{H}_{12}=\boldsymbol{H}_{22}=\begin{bmatrix}-0.2&0\\0.3&2\end{bmatrix},$$

$$\boldsymbol{T}_{11}=\begin{bmatrix}-1\\0.5\end{bmatrix},\boldsymbol{T}_{12}=\begin{bmatrix}-1\\0.2\end{bmatrix},$$

$$\boldsymbol{H}_{12}=\begin{bmatrix}-0.2&1\\0.5&3\end{bmatrix},$$

$$\boldsymbol{G}_{11}=\begin{bmatrix}3\\1\end{bmatrix},\boldsymbol{G}_{12}=\begin{bmatrix}0.5\\1.2\end{bmatrix},\boldsymbol{G}_{21}=5,\boldsymbol{G}_{22}=6,$$

$$\boldsymbol{N}_1=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},\boldsymbol{N}_2=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix}.$$

利用LMI 工具箱求解式(17)和式(18)得到

$$\varepsilon_1=31.7130,\varepsilon_2=17.6751,$$

$$\boldsymbol{X}=\begin{bmatrix}2.8219&0\\0.2237&-1.2492\end{bmatrix},$$

$$\boldsymbol{Y}_1=[-4.5472\quad -35.5814],$$

$$\boldsymbol{Y}_2=[-24.7800\quad -28.0108],$$

$$\boldsymbol{Q}_{11}=\begin{bmatrix}1.3672&3.6181\\3.6181&27.2855\end{bmatrix},$$

$$\boldsymbol{Q}_{12}=\begin{bmatrix}31.3629&0.1011\\0.1011&41.3558\end{bmatrix},$$

$$\boldsymbol{Q}_{21}=\begin{bmatrix}7.5490&2.5875\\2.5875&3.2371\end{bmatrix},$$

$$\boldsymbol{Q}_{22}=\begin{bmatrix}34.3110&-0.1203\\-0.1203&34.4327\end{bmatrix}.$$

最后计算出两个状态反馈控制器增益为

$$\boldsymbol{K}_1=[-3.8691\quad 28.4828],$$

$$\boldsymbol{K}_2=[-10.5587\quad 22.4226].$$

因此系统的状态反馈控制器为

$$\boldsymbol{u}_1(t)=[-3.8691\quad 28.4828]\boldsymbol{x}(t),$$

$$\boldsymbol{u}_2(t)=[-10.5587\quad 22.4226]\boldsymbol{x}(t).$$

5 结　语

本文考虑了带有不确定项和多时滞项的较为复杂的切换广义系统无源控制问题.通过一系列推导证明得出了使切换广义系统渐近稳定同时严格无源的充分条件.提出了鲁棒无源状态反馈控制器设计方法,并以实例说明了可行性.

参考文献:

[1] Moylan P J. Implication of passivity in a class of nonlinear systems[J]. *IEEE Transactions on Automatic Control*,1974,19(4):373-381.

[2] Byrnes C I, Lin W. Passivity and absolute stabilization of a class of discrete-time nonlinear systems[J]. *Automatica*,1995,31(2):263-267.

[3] Qin H S, Hong Y G. Passivity, optimality and stability [J]. *Control Theory and Application*,1994,11(4):421-427.

[4] Wu Z G, Park J H, Su H, et al. Dissipativity analysis for singular systems with time-varying delays[J]. *Applied Mathematics and Computation*,2011,218(8):4605-4613.

[5] 董心壮,张庆灵,赵立纯.线性广义系统的无源控制[J].生物数学学报,2004,19(2):185-187.
(Dong Xin-zhuang, Zhang Qing-ling, Zhao Li-chun. Passive control of linear singular systems [J]. *Journal of Biomathematics*,2004,19(2):185-187.)

[6] 董心壮,张庆灵.线性广义系统的输出反馈无源控制[J].东北大学学报(自然科学版),2004,25(4):310-313.
(Dong Xin-zhuang, Zhang Qing-ling. Passive control of linear singular systems via output feedback[J]. *Journal of Northeastern University(Natural Science)*,2004,25(4):310-313.)

[7] 关新平,龙承念,段广仁.离散时滞系统的鲁棒无源控制[J].自动化学报,2002,28(1):146-149.
(Guan Xin-ping, Long Cheng-nian, Duan Guang-ren. Robust passive control for discrete time-delay systems [J]. *Acta Automatica Sinica*,2002,28(1):146-149.)

[8] Boyd S, Ghaoui L E, Feron E, et al. Linear matrix inequalities in system and control theory[M]. Philadelphia:SIAM,1994.

[9] Peterson I R. A stabilization algorithm for a class uncertain linear systems[J]. *Systems & Control Letters*,1987,8(4):351-357.