

涡旋压缩机的变基圆型线研究

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摘 要: 研究了具有内、外起始角的变基圆涡旋型线, 根据展开角是否在 x 轴起始, 给出两种形式的变基圆外型线方程, 并明确给出相对应的内型线方程, 推导出偏心距的表达式, 分析了两种形式的变基圆涡旋型线方程的关系. 对两种形式的变基圆涡旋型线进行双圆弧齿头修正, 给出双圆弧修正型线方程, 分析两种情况下修正型线方程的关系, 并探究修正展角对修正齿头几何形状与压缩比的影响, 为变基圆涡旋型线的研究提供了一定的理论基础和应用参考.

关 键 词: 涡旋压缩机; 变基圆型线; 起始角; 修正展角; 双圆弧修正型线
中图分类号: TB 652 **文献标志码:** A **文章编号:** 1005-3026(2019)05-0722-06

Study on the Circular Profile of Scroll Compressors with Changing Radii

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Abstract: The scroll profile of variable base circles with internal and external starting angles was studied. Based on whether the opening angle started on x axis, the external and internal profile equations of two types of variable base circles were given. The expression of eccentric distance was deduced and the relation between two types of base circles with changing radius profile equations was analyzed. The scroll profile of variable base circles was modified in double arc and the modified profile equations of double arc were given. The relation between two types of modified profile equations was analyzed and the influences of the modified opening angles on the modified tooth profile and compression ratio were explored, which may provide a reference for further studies on variable base circle scroll profiles.

Key words: scroll compressor; base circle involute with changing radius; profile starting angle; modified angle; profile with double arc modification

涡旋式压缩机是20世纪70年代发展起来的一种高效、安静、可靠的容积式压缩机,是最具发展潜力的压缩机之一.早期的涡旋盘以等壁厚为主,其型线以圆渐开线为主,线段、正四边形等渐开线也可构成等壁厚涡旋盘^[1~5].

随着加工技术的进步以及对高性能、高效率、小体积的更高需求,涡旋盘逐渐采用变壁厚设计.Tojo等^[6~7]首先提出了以变基圆渐开线作为涡旋型线;Lee等^[8]证明了平面啮合理论可以用于

涡旋盘的设计;王君等^[9~10]论证了变基圆渐开线作为涡旋型线的可行性,给出了在外型线没有起始角时的变基圆涡旋型线的形式,并对圆渐开线型线进行双圆弧修正,推导出双圆弧修正型线方程.李雪琴等^[11]给出了外型线具有起始角时的外型线方程,而对于具有起始角的内型线方程没有明确给出.

本文基于变基圆型线展开角是否在 x 轴起始,对具有起始角的内、外型线进行研究,给出了

内、外型线的方程表达式,推导了偏心距的求解方程,分析了两种情况下型线方程之间的关系;并对两种变基圆型线方程进行双圆弧齿头修正,给出了修正型线方程.

1 构成涡旋型线的条件

1.1 平面曲线啮合原理

为了分析方便,将涡旋盘简化成平面曲线可以更好地了解涡旋机械运动.首先,引入两个平面坐标系 Π_f 和 Π_m ,两个坐标系的坐标轴相互成 180° ,如图 1 所示.坐标系 Π_m 绕 Π_f 的原点以距离 r_{ob} 做公转平动,那么平面 Π_m 的坐标原点在 Π_f 坐标系中可以表示为

$$\left. \begin{aligned} x &= r_{ob} \cdot \cos \theta, \\ y &= r_{ob} \cdot \sin \theta. \end{aligned} \right\} \quad (1)$$

式中, θ 为平面 Π_m 绕 Π_f 的旋转角度.

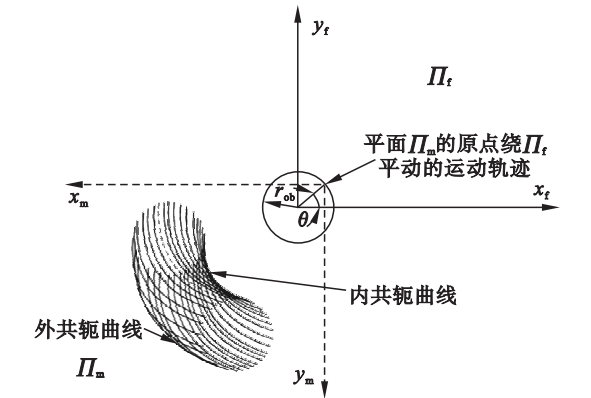


图 1 平面 Π_m 和 Π_f 所组成的平面涡旋机构
Fig. 1 Planar scroll mechanism formed by the planes Π_m and Π_f

平面中任意连续、光滑的曲线均可以用曲率中心 (x_c, y_c) 、曲率半径 ρ 和极角 ϕ 表示.那么,在平面 Π_m 中的一条连续、光滑曲线可以表示为

$$\left. \begin{aligned} x_m &= x_c(\phi_m) + \rho(\phi_m) \cdot \cos(\phi_m), \\ y_m &= y_c(\phi_m) + \rho(\phi_m) \cdot \sin(\phi_m). \end{aligned} \right\} \quad (2)$$

当平面 Π_m 逆时针绕平面 Π_f 的原点以距离 r_{ob} 做公转平动时,光滑曲线会在平面 Π_f 上形成这个曲线的线簇,进而形成内、外两条共轭曲线,如图 1 所示,内、外共轭曲线可表示为

$$\left. \begin{aligned} x_{f,i} &= -x_c(\phi_m) - [\rho(\phi_m) \pm r_{ob}] \cdot \cos(\phi_m), \\ y_{f,i} &= -y_c(\phi_m) - [\rho(\phi_m) \pm r_{ob}] \cdot \sin(\phi_m). \end{aligned} \right\} \quad (3)$$

其中, $\rho(\phi_m) - r_{ob} \geq 0$; $i = ou, in$; \pm 取正为 ou , 取负为 in .

当两个涡旋盘啮合时,啮合点一定在这内、外两条共轭曲线上,内共轭线称为内型线,外共轭线

称为外型线,又 $\phi_f = \phi_m \pm \pi$,则内、外型线的方程形式为

$$\left. \begin{aligned} x_{f,i} &= -x_c(\phi_f - \pi) - [\rho(\phi_f - \pi) \pm r_{ob}] \cdot \cos(\phi_f \pm \pi), \\ y_{f,i} &= -y_c(\phi_f - \pi) - [\rho(\phi_f - \pi) \pm r_{ob}] \cdot \sin(\phi_f \pm \pi). \end{aligned} \right\} \quad (4)$$

因此,涡旋盘内、外型线可以由曲率中心 (x_c, y_c) 、曲率半径 ρ 、偏心距 r_{ob} 和自变量 ϕ_f 表示.

1.2 变基圆渐开线

相比圆渐开线,变基圆渐开线的基圆半径是不断变化的,如图 2 所示.变基圆半径为 $a = a_0 + \delta_0 \cdot \phi$,其中 a_0 为初始半径长度, δ_0 为半径变换率, ϕ 为展开角.由渐开线性质可知,展开线增量 $d\rho$ 等于变基圆弧长的增量 dl ,即 $d\rho = dl$,则有

$$\rho = l = \int_0^\phi (a_0 + \delta_0 \cdot \phi) d\phi = a_0 \phi + \frac{\delta_0}{2} \cdot \phi^2. \quad (5)$$

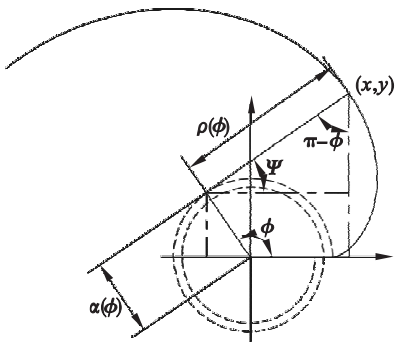


图 2 变基圆渐开线的示意图与参数定义
Fig. 2 Sketch map and parametric definitions of a circular involute with variable radii

根据基圆半径 a 、展开线 ρ 和展开角 ϕ ,可以得出变基圆渐开线方程为

$$\left. \begin{aligned} x &= a \cdot \cos \phi + \rho \cdot \sin \phi = \\ &= (a_0 + \delta_0 \cdot \phi) \cdot \cos \phi + \left(a_0 \cdot \phi + \frac{\delta_0}{2} \cdot \phi^2 \right) \cdot \sin \phi, \\ y &= a \cdot \sin \phi - \rho \cdot \cos \phi = \\ &= (a_0 + \delta_0 \cdot \phi) \cdot \sin \phi - \left(a_0 \cdot \phi + \frac{\delta_0}{2} \cdot \phi^2 \right) \cdot \cos \phi. \end{aligned} \right\} \quad (6)$$

方程(6)表示的是变基圆渐开线起始角为 0° 的情况下变基圆渐开线的方程形式,并且展开角是从 x 轴开始的.在实际设计变基圆型线时,往往渐开线都具有一定的起始角,并且展开角 ϕ 不在 x 轴起始.

2 变基圆涡旋型线

2.1 展开角在 x 轴起始

令涡旋型线为变基圆渐开线,内、外型线起始

角分别为 α_{in} 和 α_{ou} , 建立平面 Π_f 和 Π_m 分别为两个涡旋盘的坐标系, 令外型线的展开角在 x 轴起始, 则内型线起始角为 $\alpha_{in} + \alpha_{ou}$, 如图 3 所示。

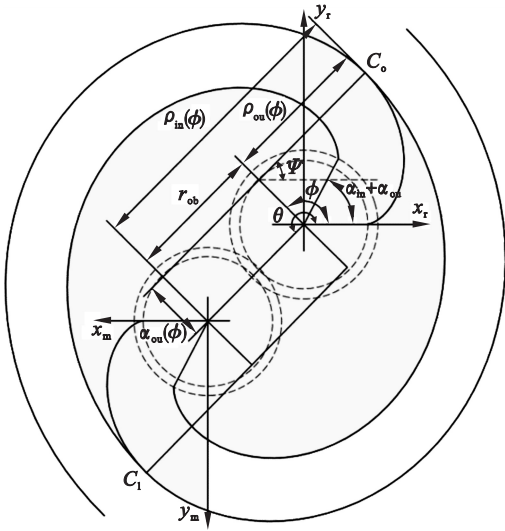


图 3 展开角在 x 轴变基圆涡旋型线示意图
Fig. 3 Scroll profile of variable base circle whose opening angle is on x -axis

在坐标系 Π_f 中, 外型线的起始角为 0° , 基圆半径起始长度为 a_0 , 基圆半径变化率为 δ_0 , 展开角为 ϕ , 则有

$$a_{f,ou} = a_0 + \delta_0 \cdot \phi, \quad (7)$$

$$\rho_{f,ou} = \int_0^\phi a_{ou} d\phi = a_0 \cdot \phi + \frac{\delta_0}{2} \cdot \phi^2. \quad (8)$$

根据变基圆渐开线方程形式, 可以得出平面 Π_f 中涡旋盘外型线方程为

$$\left. \begin{aligned} x_{f,ou} &= a_{f,ou} \cdot \cos(\phi) + \rho_{f,ou} \cdot \sin(\phi) = \\ & (a_0 + \delta_0 \cdot \phi) \cdot \cos(\phi) + (a_0 \cdot \phi + \frac{\delta_0}{2} \cdot \phi^2) \cdot \sin(\phi), \\ y_{f,ou} &= a_{f,ou} \cdot \sin(\phi) - \rho_{f,ou} \cdot \cos(\phi) = \\ & (a_0 + \delta_0 \cdot \phi) \cdot \sin(\phi) - (a_0 \cdot \phi + \frac{\delta_0}{2} \cdot \phi^2) \cdot \cos(\phi). \end{aligned} \right\} \quad (9)$$

根据平面曲线啮合理论, 一对共轭曲线具有相同的基圆半径, 可知平面 Π_m 中内型线展开角 $\phi' = \phi - (\alpha_{in} + \alpha_{ou}) + \pi$, 则基圆半径和展开线方程为

$$\left. \begin{aligned} a_{f,in} &= a_{m,ou}(\phi) = a_{m,in}(\phi') = a_0 + \delta_0 \cdot \phi = \\ & a_0 + \delta_0 \cdot [\phi' - (\pi - \alpha_{in} - \alpha_{ou})], \\ \rho_{f,in} &= \int_0^{\phi'} a_{m,in} d\phi' = \int_0^{\phi'} [a_0 + \delta_0 \cdot (\phi' - (\pi - \alpha_{in} - \alpha_{ou}))] d\phi' = \\ & \{ a_0 \cdot \phi' + \frac{\delta_0}{2} \cdot [\phi' - (\pi - \alpha_{in} - \alpha_{ou})]^2 \} \Big|_0^{\phi' = \phi + \pi - (\alpha_{in} + \alpha_{ou})} = a_0 \cdot (\phi + \pi - \alpha_{in} - \alpha_{ou}) + \end{aligned} \right\} \quad (10)$$

$$\frac{\delta_0}{2} \cdot [\phi^2 - (\pi - \alpha_{in} - \alpha_{ou})^2]. \quad (11)$$

根据方程(10), (11)可以得到坐标系 Π_f 中的内型线方程为

$$\left. \begin{aligned} x_{f,in} &= a_{f,in} \cdot \cos(\phi + \pi) + \rho_{f,in} \cdot \sin(\phi + \pi) = \\ & (a_0 + \delta_0 \cdot \phi) \cdot \cos(\phi + \pi) + [a_0 \cdot (\phi + \pi - \alpha_{in} - \alpha_{ou}) + \\ & \frac{\delta_0}{2} \cdot (\phi^2 - (\pi - \alpha_{in} - \alpha_{ou})^2)] \cdot \sin(\phi + \pi), \\ y_{f,in} &= a_{f,in} \cdot \sin(\phi + \pi) - \rho_{f,in} \cdot \cos(\phi + \pi) = \\ & (a_0 + \delta_0 \cdot \phi) \cdot \sin(\phi + \pi) - [a_0 \cdot (\phi + \pi - \alpha_{in} - \alpha_{ou}) + \\ & \frac{\delta_0}{2} \cdot (\phi^2 - (\pi - \alpha_{in} - \alpha_{ou})^2)] \cdot \cos(\phi + \pi). \end{aligned} \right\} \quad (12)$$

根据平面啮合理论, 在点 C_0 处有

$$\left. \begin{aligned} x_{f,ou} &= x_{m,in} = -x_{f,in} + r_{ob} \cdot \cos\theta, \\ y_{f,ou} &= y_{m,in} = -y_{f,in} + r_{ob} \cdot \sin\theta. \end{aligned} \right\} \quad (13)$$

式(13)中 θ 表示平面 Π_m 绕平面 Π_f 平动的公转角度, 并且 $\theta = \theta(\phi)$, 根据文献[12]可知式(13)的 Jacobian 行列式为 0, 得出 θ 与 ϕ 的关系:

$$J = \frac{\partial(x_{f,ou}, y_{f,ou})}{\partial(\phi, \theta)} = \begin{vmatrix} \frac{\partial x_{f,ou}}{\partial \phi} & \frac{\partial y_{f,ou}}{\partial \phi} \\ \frac{\partial x_{f,ou}}{\partial \theta} & \frac{\partial y_{f,ou}}{\partial \theta} \end{vmatrix} = -(\delta_0 + \rho_{in}) \cdot (\cos\phi \cdot \cos\theta + \sin\phi \cdot \sin\theta) = 0. \quad (14)$$

由方程(14)可得 $\theta = \phi + \pi/2$, 则根据方程(9), (12), (13)可以得出偏心距为

$$r_{ob} = a_0 \cdot (\pi - \alpha_{in} - \alpha_{ou}) - \frac{\delta_0}{2} \cdot (\pi - \alpha_{in} - \alpha_{ou})^2. \quad (15)$$

2.2 展开角不在 x 轴起始

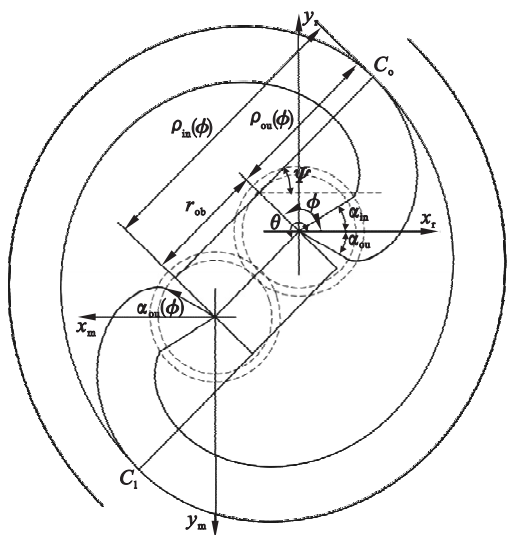
图 4 为内外起始角为 α_{in} , α_{ou} 的变基圆渐开线型线. 平面 Π_f 中外型线展开角 ϕ 在 $-\alpha_{ou}$ 处起始, 即 $\phi \in [-\alpha_{ou}, \phi]$, 则有

$$a_{f,ou} = a_0 + \delta_0 \cdot (\phi + \alpha_{ou}), \quad (16)$$

$$\begin{aligned} \rho_{f,ou} &= \int_{-\alpha_{ou}}^\phi a_{f,ou} d\phi = \int_{-\alpha_{ou}}^\phi [a_0 + \delta_0 \cdot (\phi + \alpha_{ou})] d\phi = \\ & a_0 \cdot (\phi + \alpha_{ou}) + \frac{\delta_0}{2} \cdot (\phi + \alpha_{ou})^2. \end{aligned} \quad (17)$$

那么在平面 Π_f 中, 外型线方程为

$$\left. \begin{aligned} x_{f,ou} &= a_{f,ou} \cdot \cos\phi + \rho_{f,ou} \cdot \sin\phi = \\ & [a_0 + \delta_0 \cdot (\phi + \alpha_{ou})] \cdot \cos\phi + [a_0 \cdot (\phi + \alpha_{ou}) + \\ & \frac{\delta_0}{2} \cdot (\phi + \alpha_{ou})^2] \cdot \sin\phi, \\ y_{f,ou} &= a_{f,ou} \cdot \sin\phi - \rho_{f,ou} \cdot \cos\phi = \\ & [a_0 + \delta_0 \cdot (\phi + \alpha_{ou})] \cdot \sin\phi - \\ & [a_0 \cdot (\phi + \alpha_{ou}) + \frac{\delta_0}{2} \cdot (\phi + \alpha_{ou})^2] \cdot \cos\phi. \end{aligned} \right\} \quad (18)$$

图4 展开角不在 x 轴的变基圆涡旋型线示意图Fig. 4 Scroll profile of variable base circle whose opening angle is not on x -axis

令平面 Π_m 中的内型线展开角为 $\phi' \geq 0$, 则有 $\phi' = \phi + \pi - \alpha_{in}$, 同理有

$$a_{f,in} = a_{m,ou} = a_{m,in}(\phi') = a_0 + \delta_0 \cdot (\phi + \alpha_{ou}) = a_0 + \delta_0 \cdot (\phi' - \pi + \alpha_{in} + \alpha_{ou}). \quad (19)$$

$$\rho_{f,in} = \int_0^{\phi'} a_{m,in} d\phi' = \int_0^{\phi'} [a_0 + \delta_0 \cdot (\phi' - \pi + \alpha_{in} + \alpha_{ou})] d\phi' = \left[a_0 \phi' + \frac{\delta_0}{2} \cdot (\phi' - \pi + \alpha_{in} + \alpha_{ou})^2 \right] \Big|_0^{\phi' = \phi + \pi - \alpha_{in}} = a_0 \cdot (\phi + \pi - \alpha_{in}) + \frac{\delta_0}{2} \cdot [(\phi + \alpha_{ou})^2 - (\pi - \alpha_{in} - \alpha_{ou})^2]. \quad (20)$$

根据方程(19), (20)可以得到坐标系 Π_f 中的内型线方程为

$$\left. \begin{aligned} x_{f,in} &= a_{m,in} \cdot \cos(\phi + \pi) + \rho_{m,in} \cdot \sin(\phi + \pi) = \\ &[a_0 + \delta_0 \cdot (\phi + \alpha_{ou})] \cdot \cos(\phi + \pi) + \\ &\left\{ a_0 \cdot (\phi + \pi - \alpha_{in}) + \frac{\delta_0}{2} \cdot [(\phi + \alpha_{ou})^2 - (\pi - \alpha_{in} - \alpha_{ou})^2] \right\} \cdot \sin(\phi + \pi), \\ y_{f,in} &= a_{m,in} \cdot \sin(\phi + \pi) - \rho_{m,in} \cdot \cos(\phi + \pi) = \\ &[a_0 + \delta_0 \cdot (\phi + \alpha_{ou})] \cdot \sin(\phi + \pi) - \\ &\left\{ a_0 \cdot (\phi + \pi - \alpha_{in}) + \frac{\delta_0}{2} \cdot [(\phi + \alpha_{ou})^2 - (\pi - \alpha_{in} - \alpha_{ou})^2] \right\} \cdot \cos(\phi + \pi). \end{aligned} \right\} \quad (21)$$

根据平面啮合理论, 在点 C_0 处有

$$\left. \begin{aligned} x_{f,ou} &= x_{m,in} = -x_{f,in} + r_{ob} \cdot \cos\theta, \\ y_{f,ou} &= y_{m,in} = -y_{f,in} + r_{ob} \cdot \sin\theta. \end{aligned} \right\} \quad (22)$$

则方程(22)的 Jacobian 行列式为 0, 即

$$J = \frac{\partial(x_{f,in}, y_{f,in})}{\partial(\phi, \theta)} = \begin{vmatrix} \frac{\partial x_{f,in}}{\partial \phi} & \frac{\partial y_{f,in}}{\partial \phi} \\ \frac{\partial x_{f,in}}{\partial \theta} & \frac{\partial y_{f,in}}{\partial \theta} \end{vmatrix} =$$

$$(\delta_0 + \rho_{in}) \cdot (\cos\phi \cdot \cos\theta + \sin\phi \cdot \sin\theta) = 0. \quad (23)$$

由方程(23)可得 $\theta = \phi + \pi/2$, 可得偏心距为

$$r_{ob} = a_0 \cdot (\pi - \alpha_{in} - \alpha_{ou}) - \delta_0 \cdot (\pi - \alpha_{in} - \alpha_{ou})^2 / 2. \quad (24)$$

由 2.1 和 2.2 节可知无论展开角在何处起始, 涡旋盘偏心距 r_{ob} 只与基圆初始半径 a_0 , 基圆变化率 δ_0 , 内、外型线起始角 α_{in}, α_{ou} 有关. 当涡旋盘的初始基圆半径 a_0 、基圆变化率 δ_0 、内外起始角 α_{in}, α_{ou} 一定时, 上述两种方程形式的涡旋型线如图 5 所示. 将图 5 中展开角不在 x 轴起始的型线方程绕坐标系原点逆时针旋转 α_{ou} 角度, 发现与展开角在 x 轴起始的型线方程重合. 可知方程(9), (12) 与(18), (21) 的表达形式虽然不同, 但所表达的变基圆涡旋型线相同. 因此给定基圆初始半径、基圆半径变化率和内、外渐开线起始角, 就可以唯一确定一组变基圆涡旋型线.

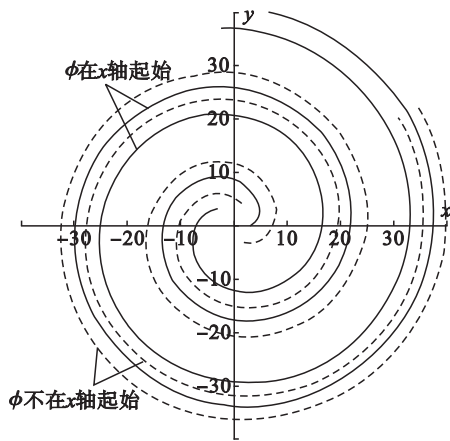


图5 两种方程形式的变基圆涡旋型线

Fig. 5 Two types of scroll profile of variable base circle equations

3 变基圆涡旋型线的齿头修正

3.1 展开角在 x 轴起始的双圆弧修正

对于初始基圆半径为 a_0 , 基圆半径变化率为 δ_0 , 内、外起始角为 α_{in}, α_{ou} 的变基圆涡旋型线的双圆弧修正步骤为: 以基圆半径中心为坐标原点 O , 令修正展角为 β , 在内、外型线上分别取中线展角为 $\beta + \pi, \beta$ 的两点 C_{in}, C_{ou} , 过点 C_{in}, C_{ou} 分别作内、外型线的法线, 过原点 O 做两条法线的垂线, 交点为 A, B 两点; 以 O 为圆心做直径为 r_{ob} 的特征圆, 线段 $C_{in}C_{ou}$ 交特征圆于 D, H 两点, 连接 OD 并双向延长, 分别交法线 AC_{ou}, BC_{in} 于 E, F 两点; 最后分别以点 E, F 为圆心, 以 EC_{ou}, FC_{in} 为半径作小圆弧 $\widehat{C_{ou}D}$, 大圆弧 $\widehat{C_{in}D}$; $\widehat{C_{ou}D}, \widehat{C_{in}D}$ 即为修正圆弧, 如图 6 所示.

由文献[9]可知,在双圆弧修正型线中点 A , B 关于原点 O 对称, $AC_{ou} // BC_{in}$, 且 BC_{in} 与 AC_{ou} 之差为 r_{ob} , 因此可以得到关系式:

$$(R_{\text{in}} + R_{\text{ou}})^2 - (2\alpha_{\text{ou}})^2 = (2d)^2, \quad (25)$$

$$d = \rho_{\text{oil}} - R_{\text{oil}}, \quad (26)$$

$$R_{\text{in}} - R_{\text{ou}} = r_{\text{ob}}. \quad (27)$$

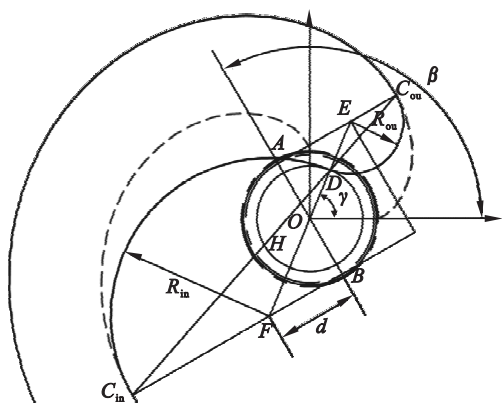


图6 展开角在 x 轴起始的双圆弧修正型线
Fig. 6 Modified double arc line of which opening angle is started on x -axis

联立方程(25),(26),(27)可得

$$d = \frac{(\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})^2 - (a_{\text{ou}})^2}{2 \cdot (\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})}, \quad (28)$$

$$R_{\text{ou}} = \rho_{\text{ou}} - \frac{(\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})^2 - (a_{\text{ou}})^2}{2 \cdot (\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})}, \quad (29)$$

$$R_{\text{in}} = r_{\text{ob}} + \rho_{\text{ou}} - \frac{(\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})^2 - (a_{\text{ou}})^2}{2 \cdot (\rho_{\text{ou}} + \frac{r_{\text{ob}}}{2})}. \quad (30)$$

又根据方程(7),(8)可知当修正展角为 β 时, a_{ou}, ρ_{ou} 为

$$a_{\text{on}} = a_0 + \delta_0 \cdot \beta, \quad (31)$$

$$\rho_{\text{ou}} = \int_0^\beta a_{\text{ou}} d\phi = a_0 \cdot \beta + \frac{\delta_0}{2} \cdot \beta^2. \quad (32)$$

继而可得修正角 γ 为

$$\gamma = \beta - \arctan\left(\frac{d}{a_{\text{on}}}\right). \quad (33)$$

则双圆弧修正方程为

$$\left. \begin{aligned} x_{\text{ou}} &= R_{\text{ou}} \cdot \cos \theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \cos \gamma; \\ y_{\text{ou}} &= R_{\text{ou}} \cdot \sin \theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \sin \gamma; \\ \theta &\in [\gamma - \pi, \beta - \frac{\pi}{2}]. \end{aligned} \right\} \quad (34)$$

$$\left. \begin{aligned} x_{\text{in}} &= R_{\text{in}} \cdot \cos\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \cos(\gamma + \pi); \\ y_{\text{in}} &= R_{\text{in}} \cdot \sin\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \sin(\gamma + \pi); \\ \theta &\in [\gamma, \beta + \frac{\pi}{2}]. \end{aligned} \right\} \quad (35)$$

双圆弧修正型线的存在需要满足条件:

- ① $R_{ou} > 0$,即保证外型线与修正圆弧光滑连接;
② $\beta > \alpha_{ou}$,即保证外圆弧与外型线有交点.

3.2 展开角不在 x 轴起始的双圆弧修正

当展开角不在 x 轴起始时, 同样给定初始基圆半径为 a_0 , 基圆半径变化率为 δ_0 , 内、外起始角为 $\alpha_{\text{in}}, \alpha_{\text{ou}}$ 的变基圆涡旋型线, 双圆弧修正型线如图 7 所示. 令 β 为修正展角, 则有 $\phi_\beta = \beta - \alpha_{\text{ou}}$, 修正角 $\phi_\gamma = \beta - \alpha_{\text{ou}} - \arctan(d/a_{\text{ou}})$, 对应的基圆半径和展开线为

$$a_{\text{ou}} = a_0 + \delta_0 \cdot (\phi_\beta + \alpha_{\text{ou}}), \quad (36)$$

$$\rho_{\text{ou}} = \int_0^\beta a_{\text{ou}} d\phi = a_0 \cdot (\phi_\beta + \alpha_{\text{ou}}) + \frac{\delta_0}{2} \cdot (\phi_\beta + \alpha_{\text{ou}})^2. \quad (37)$$

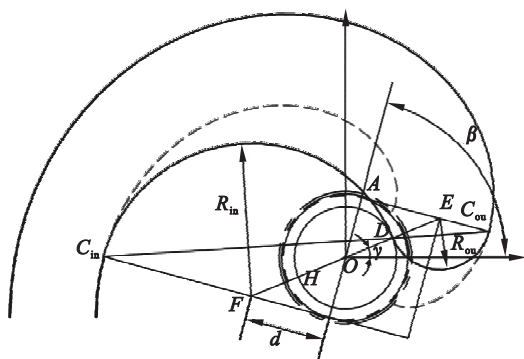


图 7 展开角不在 x 轴起始的双圆弧齿头修正型线
Fig. 7 Modified double arc line of which opening angle is not started on x -axis

由图 7 可知 R_{ou} , R_{in} 和 d 的关系式同样如方程 (25), (26) 和 (27) 所示, 则修正角为

$$\phi_\gamma = \beta - \alpha_{\text{ou}} - \arctan(d/a_{\text{ou}}). \quad (38)$$

因此,展开角不在 x 轴起始的双圆弧修正方程为

$$\left. \begin{aligned} x_{\text{ou}} &= R_{\text{ou}} \cdot \cos\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \cos\gamma; \\ y_{\text{ou}} &= R_{\text{ou}} \cdot \sin\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \sin\gamma; \\ \theta &\in [\phi_\gamma - \pi, \beta - \frac{\pi}{2}]. \end{aligned} \right\} \quad (39)$$

$$\left. \begin{aligned} x_{\text{in}} &= R_{\text{in}} \cdot \cos\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \cos(\gamma + \pi); \\ y_{\text{in}} &= R_{\text{in}} \cdot \sin\theta + \sqrt{a_{\text{ou}}^2 + d^2} \cdot \sin(\gamma + \pi); \\ \theta &\in [\phi_\gamma, \beta + \frac{\pi}{2}] \end{aligned} \right\} \quad (40)$$

对于不同修正展角, 涡旋齿头变化如图 8 所

示.在图 8 中修正展角由 $\pi/4$ 逐渐增大到 $13\pi/12$,涡旋齿头的壁厚逐渐增厚,长度逐渐减小.当修正展角较小时,压缩气体到排气口需要动涡盘公转平动更大的角度,能够获得更高的压缩比.

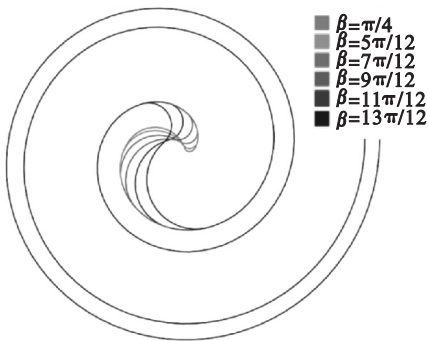


图 8 不同修正展角的双圆弧修正型线
Fig. 8 Different β values to the scroll profile with arc modification

4 结 论

- 1) 针对变基圆涡旋型线展开角是否在 x 轴起始的两种情况,推导出具有起始角的外型线方程.根据平面曲线啮合原理推导出具有起始角的内型线方程,在此基础上得到两种形式的涡旋型线方程.
- 2) 对比两种形式的涡旋型线方程,在基圆初始半径、基圆半径变化率和内、外型线起始角一定时,有相同的变基圆涡旋型线,可依据设计加工需求选择适合的型线方程形式.涡旋盘的偏心距由基圆初始半径、基圆半径变化率和内、外型线起始角确定,给出偏心距的求解方程.
- 3) 采用双圆弧修正法对两种形式的变基圆型线方程进行齿头修正,给出了双圆弧齿头修正方程.当修正展角增大时,涡旋齿头的壁厚增加、长度减小,适当选取较小的修正展角,可以获得更大的压缩行程,提高压缩比.

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