

带有变号格林函数的二阶边值问题正解

张国伟, 曲雪冰

(东北大学 理学院, 辽宁 沈阳 110819)

摘 要: 研究了一类带有变号格林函数的二阶边值问题正解的存在性, 格林函数变号由边值条件中系数的不同取值所致, 这与文献中通常由未知函数一次项系数的变化导致格林函数变号不同. 没有非线性项非负的限制时, 通过对格林函数的正部和负部赋予约束条件, 证明了二阶边值问题正解的存在性. 利用两个具体例子说明了理论结果的有效性, 例子中边值条件的系数包含了正的和负的两种情形. 另外对两类不同的边值条件给出了说明.

关 键 词: 正解; 变号格林函数; 二阶边值问题; 全连续算子; Leray-Schauder 不动点定理

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Positive Solutions of Second-Order Boundary Value Problems with Sign-Changing Green's Function

ZHANG Guo-wei, QU Xue-bing

(School of Sciences, Northeastern University, Shenyang 110819, China. Corresponding author: QU Xue-bing, E-mail: 2865282169@qq.com)

Abstract: The existence of positive solutions for a class of second-order boundary value problems with a sign-changing Green's function was studied, and the sign-changing Green's function was caused by different values of coefficients in boundary value conditions, which is different from that the change of the coefficient of the first order of the unknown function usually leads to the change of the Green's function. When there is no non-negative limitation of nonlinear term, the existence of positive solutions for second-order boundary value problems was proved by giving constraints to the positive and negative parts of Green's function. The validity of the theoretical results was illustrated by two concrete examples, in which the coefficients of boundary value condition include both positive and negative cases. In addition, two different boundary conditions were explained.

Key words: positive solution; sign-changing Green's function; second-order boundary value problem; completely continuous operator; Leray-Schauder fixed point theorem

Torres^[1] 讨论了一类线性周期边值问题的格林函数, 在一定条件下证明了其格林函数是不变号的. Cabada 等^[2] 假设格林函数是正的情形时, 证明了非线性周期边值问题正解的存在性. 其他一些相关的工作可参见文献[3-8]. Graef 等^[9] 和 Ma^[10] 分别讨论了在格林函数非负和变号情形下非线性周期边值问题正解的存在性. Zhong 等^[11] 讨论了当 $0 < m \leq \frac{3}{4}$ 时, 非线性周期边值问题式(1)的正解存在性:

$$\left. \begin{aligned} y'' + m^2 y &= f(y), & 0 \leq t \leq 2\pi; \\ y(0) &= y(2\pi), & y'(0) = y'(2\pi). \end{aligned} \right\} \quad (1)$$

实际上, 当 $\frac{1}{2} < m \leq \frac{3}{4}$ 时, 其格林函数是变号的.

特别地, Gao 等^[12] 研究了具有变号格林函数非线性周期边值问题式(2)的正解存在性:

$$\left. \begin{aligned} y'' + \left(\frac{1}{2} + \varepsilon \right) y &= f(y), & 0 \leq t \leq 2\pi; \\ y(0) &= y(2\pi), & y'(0) = y'(2\pi). \end{aligned} \right\} \quad (2)$$

式中, $0 < \varepsilon < \frac{1}{2}$. 考虑边值条件含有参数的非线性边值问题式(3)的正解存在性:

$$\left. \begin{aligned} y'' + \frac{1}{4}y &= \lambda g(t)f(y), & 0 \leq t \leq 2\pi; \\ y(0) &= \alpha y(2\pi), & y'(0) = \beta y'(2\pi). \end{aligned} \right\} \quad (3)$$

其中格林函数的变号性是由参数不同取值导致的, 结论的证明方法来自文献[12].

$C[0, 2\pi]$ 表示 $[0, 2\pi]$ 上连续函数的 Banach 空间, 范数为 $\|y\|_0 = \max_{t \in [0, 2\pi]} |y(t)|, \forall y \in C[0, 2\pi]$.

1 格林函数及其变号性

首先假设:
 H_1 : $\alpha \neq \beta, \alpha \neq 0, \beta \neq 0, \alpha \neq \pm 1, \beta \neq \pm 1$;
 H_2 : $f \in C[0, \infty)$ 是连续的, 且 $f(0) > 0$;
 H_3 : $g \in L^1[0, 2\pi]$, 对于任意区间不是几乎处处等于零.

将 $f(x)$ 再进行如下连续延拓: 当 $x < 0$ 时, 规定 $f(x) = f(0)$, 仍记为 $f(x)$. 定义算子
 $(Sy)(t) = \lambda \int_0^{2\pi} G(t, s)g(s)f(y)ds, t \in [0, 2\pi]$.
 式中,

$$G(t, s) = \frac{\alpha - 1}{\alpha + 1} \cos\left(\frac{t}{2}\right) \sin\left(\frac{s}{2}\right) + \frac{1 - \beta}{1 + \beta} \sin\left(\frac{t}{2}\right) \cos\left(\frac{s}{2}\right) + \sin\left(\frac{|t - s|}{2}\right)$$

是边值问题(3)的格林函数. 利用常规方法可以证明下面的引理.

引理 1 设 H_1, H_2 和 H_3 满足. 边值问题式(3)存在解 $y \in C[0, 2\pi]$, 等价于 y 是算子 S 在空间 $C[0, 2\pi]$ 的不动点.

定理 1 设 H_1 满足, 则格林函数 $G(t, s)$ 在 $[0, 2\pi] \times [0, 2\pi]$ 变号.

证明 根据参数 α 与 β 的关系, 可划分为如下 20 个参数区域 I – XX, 见图 1.

仅给出对于区域 $0 < \beta < \alpha$ 的讨论, 其余区域的讨论方法类似.

1) $\pi > t > s > 0$ 情形: 因为

$$G(t, s) = \frac{2}{(1 + \alpha)(1 + \beta)} \left[(1 + \alpha) \sin\left(\frac{t}{2}\right) \cos\left(\frac{s}{2}\right) - (1 + \beta) \cos\left(\frac{t}{2}\right) \sin\left(\frac{s}{2}\right) \right],$$

$$G(t, s) > 0 \Leftrightarrow \frac{1 + \alpha}{1 + \beta} \tan\left(\frac{t}{2}\right) > \tan\left(\frac{s}{2}\right) \Leftrightarrow s < 2\arctan\left[\frac{1 + \alpha}{1 + \beta} \tan\left(\frac{t}{2}\right)\right],$$

故恒有 $G(t, s) > 0$. 事实上, $\frac{1 + \alpha}{1 + \beta} > 1$,

$$2\arctan\left[\frac{1 + \alpha}{1 + \beta} \tan\left(\frac{t}{2}\right)\right] > 2\arctan\left[\tan\left(\frac{t}{2}\right)\right] = t > s.$$

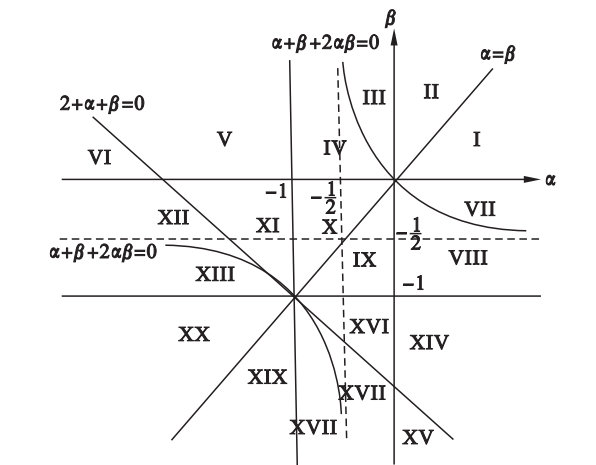


图 1 参数区域
 Fig. 1 Domain of parameters

2) $\pi > s > t > 0$ 情形: 因为

$$G(t, s) = \frac{2}{(1 + \alpha)(1 + \beta)} \left[\alpha(1 + \beta) \sin\left(\frac{s}{2}\right) \cos\left(\frac{t}{2}\right) - \beta(1 + \alpha) \cos\left(\frac{s}{2}\right) \sin\left(\frac{t}{2}\right) \right],$$

$$G(t, s) > 0 \Leftrightarrow \tan\left(\frac{s}{2}\right) > \frac{\beta(1 + \alpha)}{\alpha(1 + \beta)} \tan\left(\frac{t}{2}\right) \Leftrightarrow s > 2\arctan\left[\frac{\beta(1 + \alpha)}{\alpha(1 + \beta)} \tan\left(\frac{t}{2}\right)\right],$$

故恒有 $G(t, s) > 0$. 事实上, $0 < \frac{\beta(1 + \alpha)}{\alpha(1 + \beta)} < 1$,

$$2\arctan\left[\frac{\beta(1 + \alpha)}{\alpha(1 + \beta)} \tan\left(\frac{t}{2}\right)\right] < 2\arctan\left[\tan\left(\frac{t}{2}\right)\right] = t < s.$$

3) $2\pi > s > \pi > t > 0$ 情形: 因为 $G(t, s)$ 与情形 2) 相同, 所以

$$G(t, s) > 0 \Leftrightarrow \tan\left(\frac{s}{2}\right) < \frac{\beta(1 + \alpha)}{\alpha(1 + \beta)} \tan\left(\frac{t}{2}\right).$$
 由于 $\tan\left(\frac{s}{2}\right) < 0, \tan\left(\frac{t}{2}\right) > 0$, 故 $G(t, s) > 0$ 恒成立.

4) $2\pi > t > \pi > s > 0$ 情形: 因为 $G(t, s)$ 与情形 1) 相同, 所以

$$G(t, s) > 0 \Leftrightarrow \frac{1 + \alpha}{1 + \beta} \tan\left(\frac{t}{2}\right) < \tan\left(\frac{s}{2}\right).$$
 由于 $\tan\left(\frac{t}{2}\right) < 0, \tan\left(\frac{s}{2}\right) > 0$, 故 $G(t, s) > 0$ 恒成立.

5) $2\pi > t > s > \pi$ 情形: 因为 $G(t, s)$ 与情形 1) 相同, 所以

$$G(t, s) > 0 \Leftrightarrow \frac{1+\alpha}{1+\beta} \tan\left(\frac{t}{2}\right) > \tan\left(\frac{s}{2}\right) \Leftrightarrow s < 2\pi + 2\arctan\left[\frac{1+\alpha}{1+\beta} \tan\left(\frac{t}{2}\right)\right].$$

令 $s = 2\pi + 2\arctan\left[\frac{1+\alpha}{1+\beta} \tan\left(\frac{t}{2}\right)\right]$.

由于 $\frac{1+\alpha}{1+\beta} > 1$, $\tan\left(\frac{t}{2}\right) < 0$, 则

$$s' = \left\{ 2\pi + 2\arctan\left[\frac{1+\alpha}{1+\beta} \tan\left(\frac{t}{2}\right)\right] \right\}' = \frac{\frac{1+\alpha}{1+\beta} \sec^2\left(\frac{t}{2}\right)}{1 + \left(\frac{1+\alpha}{1+\beta}\right)^2 \tan^2\left(\frac{t}{2}\right)} > 0,$$

$$s'' = \left\{ 2\pi + 2\arctan\left[\frac{1+\alpha}{1+\beta} \tan\left(\frac{t}{2}\right)\right] \right\}'' = \frac{\frac{1+\alpha}{1+\beta} \left[1 - \left(\frac{1+\alpha}{1+\beta}\right)^2 \right] \sec^2\left(\frac{t}{2}\right) \tan\left(\frac{t}{2}\right)}{\left[1 + \left(\frac{1+\alpha}{1+\beta}\right)^2 \tan^2\left(\frac{t}{2}\right) \right]^2} > 0,$$

且当 $t \rightarrow \pi$ 时, $s \rightarrow \pi$; 当 $t \rightarrow 2\pi$ 时, $s \rightarrow 2\pi$. 故由单调性与凹凸性可知其符号.

6) $2\pi > s > t > \pi$ 情形: 因为 $G(t, s)$ 与情形 2) 相同, 所以

$$G(t, s) > 0 \Leftrightarrow \tan\left(\frac{s}{2}\right) > \frac{\beta(1+\alpha)}{\alpha(1+\beta)} \tan\left(\frac{t}{2}\right) \Leftrightarrow s > 2\pi + 2\arctan\left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)} \tan\left(\frac{t}{2}\right)\right].$$

令 $s = 2\pi + 2\arctan\left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)} \tan\left(\frac{t}{2}\right)\right]$, 由于

$0 < \frac{\beta(1+\alpha)}{\alpha(1+\beta)} < 1$, $\tan\left(\frac{t}{2}\right) < 0$, 则

$$s' = \left\{ 2\pi + 2\arctan\left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)} \tan\left(\frac{t}{2}\right)\right] \right\}' = \frac{\frac{\beta(1+\alpha)}{\alpha(1+\beta)} \sec^2\left(\frac{t}{2}\right)}{1 + \left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)}\right]^2 \tan^2\left(\frac{t}{2}\right)} > 0,$$

$s'' =$

$$\frac{\frac{\beta(1+\alpha)}{\alpha(1+\beta)} \left\{ 1 - \left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)}\right]^2 \right\} \sec^2\left(\frac{t}{2}\right) \tan\left(\frac{t}{2}\right)}{\left\{ 1 + \left[\frac{\beta(1+\alpha)}{\alpha(1+\beta)}\right]^2 \tan^2\left(\frac{t}{2}\right) \right\}^2} < 0,$$

且当 $t \rightarrow \pi$ 时, $s \rightarrow \pi$; 当 $t \rightarrow 2\pi$ 时, $s \rightarrow 2\pi$. 故由单调性与凹凸性可知其符号.

综上所述, 对于区域 $0 < \beta < \alpha$ 来说, 格林函数符号情况可见图 2.

2 边值问题的正解

对于 $t \in [0, 2\pi]$, 记

$$\begin{aligned} [G(t, s)g(s)]^+ &= \\ \begin{cases} G(t, s)g(s), & G(t, s)g(s) \geq 0; \\ 0, & G(t, s)g(s) < 0; \end{cases} \\ [G(t, s)g(s)]^- &= \\ \begin{cases} -G(t, s)g(s), & G(t, s)g(s) \leq 0; \\ 0, & G(t, s)g(s) > 0. \end{cases} \end{aligned}$$

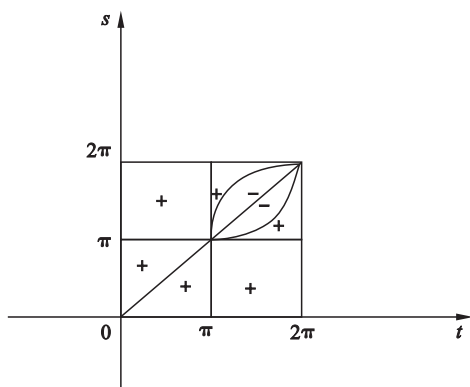


图 2 $0 < \beta < \alpha$ 时格林函数的符号

Fig. 2 Signs of Green's function for $0 < \beta < \alpha$

再假设

H_4 : 存在 $\varepsilon > 0$, 使得对于 $t \in [0, 2\pi]$,

$$\int_0^{2\pi} \{ [G(t, s)g(s)]^+ - (1 + \varepsilon) \times [G(t, s)g(s)]^- \} ds \geq 0.$$

引理 2 如果 H_1, H_3 和 H_4 满足, 令

$$p(t) = \int_0^{2\pi} [G(t, s)g(s)]^+ ds, \text{ 则 } \|p\|_0 > 0.$$

证明 如果 $p(t) \equiv 0$, 则 $\forall t \in [0, 2\pi]$, 有 $[G(t, s)g(s)]^+ = 0$, a. e. $s \in [0, 2\pi]$. 由 H_4 可知, $\forall t \in [0, 2\pi]$, $[G(t, s)g(s)]^- = 0$, a. e. $s \in [0, 2\pi]$. 故 $G(t, s)g(s) = 0$, a. e. $s \in [0, 2\pi]$. 从而由定理 1 可知, 存在使 g 几乎处处等于零的区间, 这与 H_3 矛盾. 证毕.

定义算子 T 为

$$(Ty)(t) = \lambda \int_0^{2\pi} [G(t, s)g(s)]^+ f[y(s)] ds,$$

$\forall y \in C[0, 2\pi]$. 易证 $T: C[0, 2\pi] \rightarrow C[0, 2\pi]$ 是全连续算子. 利用引理 2, 可得如下引理.

引理 3 如果 $H_1 \sim H_4$ 满足, 令 $0 < \delta < 1$, 则存在正数 $\bar{\lambda}$, 使得当 $\lambda \in (0, \bar{\lambda})$ 时, 方程 $y(t) = (Ty)(t)$ 有一个正解 \hat{y}_λ , 满足当 $\lambda \rightarrow 0$ 时, $\|\hat{y}_\lambda\|_0 \rightarrow 0$. 并且 $\hat{y}_\lambda \geq \lambda \delta f(0)p(t)$, $t \in [0, 2\pi]$.

定理 2 如果 $H_1 \sim H_4$ 满足, 则存在 $\lambda_0 > 0$, 使得当 $\lambda \in (0, \lambda_0)$ 时, 式(3)有一个正解.

证明 令 $q(t) = \int_0^{2\pi} [G(t,s)g(s)]^- ds$. 由 H_4 可知, 如果 $q(t) \neq 0$, 由于 $\varepsilon > 0$, 取 ξ 满足 $\frac{1}{1+\varepsilon} < \xi < 1$, 故 $(1+\varepsilon)\xi > 1$, 于是 $\xi f(0) \frac{p(t)}{q(t)} \geq \xi f(0)(1+\varepsilon) > f(0)$. 又因为 f 在 0 处连续, 存在 $k \in (0, 1)$, 使得对 $s \in [0, k]$, 有 $|f(s)| \leq \xi f(0)(1+\varepsilon)$. 所以

$$q(t) |f(s)| \leq \xi p(t) f(0). \tag{4}$$

如果 $q(t) = 0$, 式(4)成立.
设 $\delta \in (\xi, 1)$, 由引理 3 可知 $\lim_{\lambda \rightarrow 0^+} (\|\tilde{y}_\lambda\|_0 + \lambda \delta f(0) \|p\|_0) = 0 < k$, 所以存在 $\lambda_1 > 0$, 使得当 $\lambda \in (0, \lambda_1)$ 时, $\|\tilde{y}_\lambda\|_0 + \lambda \delta f(0) \|p\|_0 \leq k$. 因为 f 在 $[-k, k]$ 上一致连续, 故对 $f(0) \left(\frac{\delta-\xi}{2}\right) > 0$, 存在 $\lambda_2 > 0$, 使得当 $x, y \in [0, k]$ 且 $|x-y| \leq \lambda_2 \delta f(0) \|p\|_0$ 时, 有 $|f(x) - f(y)| \leq f(0) \left(\frac{\delta-\xi}{2}\right)$. 取 $\lambda_0 = \min\{\lambda_1, \lambda_2\}$, 当 $\lambda \in (0, \lambda_0)$ 时,

$$\|\tilde{y}_\lambda\|_0 + \lambda \delta f(0) \|p\|_0 \leq k. \tag{5}$$

当 $x, y \in [-k, k]$, 且 $|x-y| \leq \lambda_0 \delta f(0) \|p\|_0$ 时, $|f(x) - f(y)| \leq f(0) \left(\frac{\delta-\xi}{2}\right)$. $\tag{6}$

当 $\lambda \in (0, \lambda_0)$ 时, 定义算子 H 如下:
 $(Hy)(t) = \lambda \int_0^{2\pi} G(t,s)g(s)f(\tilde{y}_\lambda + y)ds - \tilde{y}_\lambda(t)$, $t \in [0, 2\pi]$, 其中 \tilde{y}_λ 由引理 3 给出. 容易证明 $H: C[0, 2\pi] \rightarrow C[0, 2\pi]$ 是全连续的.

设 $\theta \in (0, 1)$, $y \in C[0, 2\pi]$, 若 $y = \theta Hy$, 下证 $\|y\|_0 \neq \lambda \delta f(0) \|p\|_0$. 否则

$\|y\|_0 = \lambda \delta f(0) \|p\|_0$, 则由式(5)和式(6)得 $\|\tilde{y}_\lambda + y\|_0 \leq \|\tilde{y}_\lambda\|_0 + \|y\|_0 \leq k$, $\tag{7}$

$$|f(\tilde{y}_\lambda + y) - f(\tilde{y}_\lambda)| \leq f(0) \left(\frac{\delta-\xi}{2}\right). \tag{8}$$

由式(4), 式(7)和式(8)可知:
 $|y| = \theta \left| \lambda \int_0^{2\pi} G(t,s)g(s)f(\tilde{y}_\lambda + y)ds - \lambda \int_0^{2\pi} [G(t,s)g(s)]^+ f(\tilde{y}_\lambda)ds \right| \leq \lambda \int_0^{2\pi} [G(t,s)g(s)]^+ |f(\tilde{y}_\lambda + y) - f(\tilde{y}_\lambda)|ds + \lambda \int_0^{2\pi} [G(t,s)g(s)]^- \times |f(\tilde{y}_\lambda + y)|ds \leq \lambda \frac{\delta-\xi}{2} f(0) p(t) +$

$$\begin{aligned} &\lambda \left(\max_{s \in [-k, k]} |f(s)| \right) \times q(t) = \\ &\lambda \frac{\delta-\xi}{2} f(0) p(t) + \\ &\lambda \left(\max_{s \in [0, k]} |f(s)| \right) q(t) \leq \\ &\lambda \frac{\delta-\xi}{2} f(0) p(t) + \lambda \xi f(0) p(t) = \\ &\lambda \frac{\delta+\xi}{2} f(0) p(t). \end{aligned} \tag{9}$$

故与 $\|y\|_0 \leq \lambda \frac{\delta+\xi}{2} f(0) \|p\|_0 < \lambda \delta f(0) \|p\|_0$ 矛盾. 由 Leray - Schauder 不动点定理知 H 有一个不动点 \tilde{y}_λ , 且 $\|\hat{y}_\lambda\|_0 \leq \lambda \delta f(0) \|p\|_0$. 类似的推导可知 \hat{y}_λ 满足式(9). 记 $y_\lambda = \tilde{y}_\lambda + \hat{y}_\lambda$, 于是 y_λ 是式(3)的一个解. 再由引理 3 得

$$\begin{aligned} y_\lambda(t) &\geq \tilde{y}_\lambda(t) - |\hat{y}_\lambda(t)| \geq \lambda \delta f(0) p(t) - \\ &\lambda \frac{\delta+\xi}{2} f(0) p(t) = \lambda \frac{\delta-\xi}{2} f(0) p(t), \end{aligned}$$

可见 y_λ 是式(3)的一个正解.

3 例子

例 1 考虑边值问题:

$$\begin{aligned} y'' + \frac{1}{4}y &= \lambda \left(\text{sint} + \frac{1}{2} \right) \text{cos}y, t \in [0, 2\pi]; \\ y(0) &= y(2\pi), y'(0) = \left(-\frac{3}{4} \right) y'(2\pi). \end{aligned}$$

可以验证定理 2 的条件满足, 所以存在 $\lambda_0 > 0$, 当 $0 < \lambda < \lambda_0$ 时有正解.

例 2 考虑边值问题:

$$\begin{aligned} y'' + \frac{1}{4}y &= \lambda \left(\text{ecos} \left(\frac{t}{2} \right) + \frac{2}{3} \pi \right) \times \\ &(2 - e^y), t \in [0, 2\pi]; \\ y(0) &= y(2\pi), y'(0) = \frac{1}{2} y'(2\pi). \end{aligned}$$

可以验证定理 2 的条件满足, 所以存在 $\lambda_0 > 0$, 当 $0 < \lambda < \lambda_0$ 时有正解.

这里含导数部分边值的系数分别取正数和负数, 并且 $\text{sint} + \frac{1}{2}$ 和 $\text{ecos} \left(\frac{t}{2} \right) + \frac{2}{3} \pi$ 在 $t \in [0, 2\pi]$ 上都是变号的.

4 结论

本文主要研究的是由边值条件中系数 α, β 不同取值导致格林函数变号的二阶非线性边值问题, 其中 $\alpha \neq \beta$. 如果 $H_1 \sim H_4$ 满足, 则存在 $\lambda_0 > 0$, 使得当 $\lambda \in (0, \lambda_0)$ 时, 式(3)有一个正解. 当 $\alpha = \beta = 0$ 时, 其格林函数不变号. 而当 $\alpha = \beta = 1$

时,实际上是周期边值问题,其格林函数也是不变号的.

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3 结 语

1) 本文将结构方程模型和序关系分析法相结合,构建了 SEM – G 评价模型. 中国 30 省份技术创新能力差距显著, 尽管大多数省份的技术创新能力得到了提升, 但增长幅度不足导致创新能力的发展态势不甚理想; 创新投入、创新产出和创新环境之间的水平差距异常明显, 羸弱的创新要素均衡性限制了创新能力提升的可延续性.

2) 基于评价结果, 提出以下建议: 一方面地方政府应当统筹资源, 从创新环境、投入、产出等不同环节入手, 为提升创新能力提供良好的保障措施和环境支撑; 另一方面, 政府应当搭建良好的创新服务平台, 为创新知识的流通创造良好条件, 从而实现创新要素的均衡协调.

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