

基于迭代学习算法的机械臂系统轨迹跟踪控制

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摘 要: 针对平面二自由度机械臂这一非线性系统, 设计了带初态学习的指数变增益 D 型迭代学习律, 并给出收敛性证明. 仿真结果表明, 迭代学习控制对于诸如二自由度机械臂系统这类具有重复运动性质的被控对象具有很好的控制效果. 设计带初态学习的指数变增益 D 型学习律, 系统不仅在存在初态偏移的情况下实现了机械臂期望轨迹的完全跟踪, 还加快了收敛速度, 增强了迭代学习控制的鲁棒性.

关 键 词: 二自由度机械臂; 迭代学习控制; D 型学习律; 初态学习; 指数变增益

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Trajectory Tracking Control of Robotic Manipulator System Based on Iterative Learning Algorithm

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Abstract: An improved D-type learning law with time-varying exponential gain and initial state learning was proposed and the certification of convergence for this learning law was given for the planar 2-DOF manipulator which is an non-linear system. Simulation results showed that the iterative learning control was very effective for the planar 2-DOF manipulator system which owned property of repetitiveness. D-type learning law with time-varying exponential gain and initial state learning was obtained which could eliminate the impact of the initial state migration on tracking performance and improve the convergence rates of the algorithm. What's more, the robustness of iterative learning control was also improved.

Key words: 2-DOF manipulator; iterative learning control (ILC); D-type learning law; initial state learning; time-varying exponential gain

对于具有重复运动性质的非线性系统, 如机械臂的重复操作, 迭代学习控制是解决这一类系统简单而有效的方法^[1]. 因而迭代学习控制自提出以来, 就被广泛应用到机械臂的控制当中^[2-3]. 但多数迭代学习控制算法要求每次迭代的初始状态与期望轨迹对应的初始状态严格一致^[4], 而实际的重复定位操作往往会因其迭代初态相对于期望初态的偏移, 使得在理论上证明是收敛的算法, 有可能在实际中不收敛, 进而影响迭代学习控制的跟踪性能.

D 型学习律是一种基本的迭代学习律, 最早由 Arimoto 等^[1]提出, 但迭代过程中要求系统在

每次迭代时的初始状态在期望初态上, 或在期望初态某一很小的邻域内. 而实际环境下不可避免存在初态偏移, 随着迭代次数的增加, 偏差不可消除. 为此, 文献^[5]最早讨论了初始定位误差对系统跟踪性能的影响. 任雪梅等^[6]采用初始状态学习方法解决了仿射非线性系统在 D 型学习律作用下的初态问题. 李世平等^[7]针对非仿射非线性系统, 提出初态学习律, 并给出关于初态学习的收敛性充分条件. 带有初态学习的 D 型迭代学习控制算法, 虽然很好地解决了初态偏移所造成的系统输出不能完全跟踪期望轨迹的问题, 但是收敛速度却比原来下降很多. 文献^[8]提出指数变增

益加速学习算法,大大提高了学习收敛速度. 本文结合指数变增益学习算法在收敛速度上的优势,设计了一种带有初态学习的指数变增益 D 型迭代学习控制算法,并利用算子理论证明了该算法的收敛性,获得了该算法收敛的谱半径形式的充分条件. 该算法不仅解决了初态偏移所造成的系统输出不能完全跟踪期望轨迹的问题,而且其收敛速度也得到了很大的提高. 将该算法应用到平面二自由度机械臂的跟踪控制中,验证了所给算法的有效性.

1 带初态学习的指数变增益 D 型学习律设计

考虑如下的具有重复运行性质的一类非线性时变系统:

$\dot{x}(t) = f(x(t)) + B(t)u(t), y(t) = C(t)x(t).$ (1)

式中: $t \in [0, T]$; 系统状态向量 $x(t) \in \mathbf{R}^n$; 控制向量 $u(t) \in \mathbf{R}^r$; 输出向量 $y(t) \in \mathbf{R}^m$; f, B, C 为适当维数的向量或矩阵.

假设系统(1)在有限时间区间 $t \in [0, T]$ 的任意时刻都满足如下假设条件:

- 1) $f(x(t))$ 关于 $x(t)$ 满足 Lipschitz 条件,即存在 $L > 0$ 的常数,使得当 $x_1(t), x_2(t) \in \mathbf{R}^n, t \in [0, T]$, 有
- $\|f(x_1(t)) - f(x_2(t))\| \leq L \|x_1(t) - x_2(t)\|$;
- 2) 期望轨迹 $y_d(t)$ 在 $t \in [0, T]$ 上连续;
- 3) 在 $t \in [0, T]$ 上, $B(t), C(t)$ 有界;
- 4) 存在唯一理想的控制 $u_d(t)$, 使得系统的状态和输出为期望值;
- 5) $I + \lambda(t)C(t)B(t)\Gamma, \forall t \in [0, T]$ 存在逆矩阵; 其中: Γ 为定常增益矩阵; $\lambda(t)$ 为变增益系数.

该系统每次迭代时的初态都不相同, 设第 k 次迭代的初态为 $x_k(0)$, 则输出误差为

$$e_k(t) = y_d(t) - y_k(t).$$
 (2)

输入采用指数变增益的 D 型迭代学习控制律

$$u_{k+1}(t) = u_k(t) + \lambda(t) \cdot \Gamma \dot{e}_k(t).$$
 (3)

指数变增益系数 $\lambda(t) = e^{nt} (n > 0)$. 同时初始状态采用可变增益的迭代学习律

$$x_{k+1}(0) = x_k(0) + \lambda(0)B(0)\Gamma e_k(0).$$
 (4)

图 1 为带初态学习的 D 型学习律的控制结构框图, 当前的控制输入是由前一次的控制输入与误差信号的导数信号通过学习律得到的, 每一次重复操作结束时, 再依据学习律计算新的控制量存入存储器, 刷新旧的控制量.

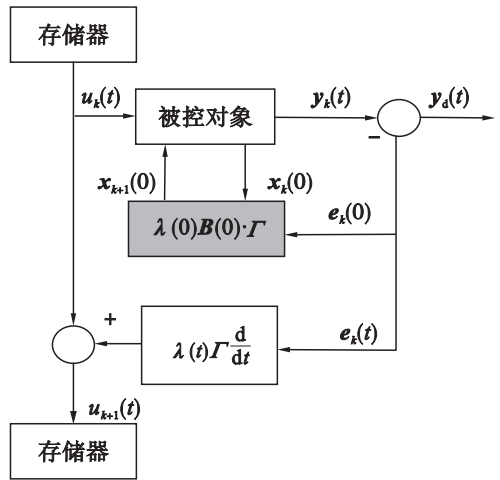


图 1 带初态学习的指数变增益 D 型学习律控制框图
Fig. 1 Control structure of time-varying exponential gain D-type learning law with initial state study

2 收敛性分析

引理 1^[9] 设 $x(t), z(t)$ 和 $b(t)$ 是 $[0, T]$ 上的实值连续函数, 且 $b(t)$ 在 $[0, T]$ 上非负, 如果 $x(t) \leq z(t) \leq \int_0^t b(\tau)x(\tau)d\tau, t \in [0, T]$, 则

$$x(t) \leq z(t) + \int_0^t b(\tau)x(\tau)e^{\int_\tau^t b(\sigma)d\sigma}d\tau.$$

引理 2^[9] 假定算子 $Q: C_r[0, T] \rightarrow C_r[0, T]$ 满足下述条件:

1) $\|Q(x)(t)\| \leq N(q + \int_0^t \|x(s)\|ds).$

$\forall x \in C_r[0, T], t \in [0, T].$

2) $\|Q(x)(t) - Q(y)(t)\| \leq N \int_0^t \|x(s) - y(s)\|ds, \forall x, y \in C_r[0, T], t \in [0, T].$

式中: N, q 为非负数, 则有

1) $\forall y \in C_r[0, T]$, 存在唯一的 $\forall x \in C_r[0, T]$, 使得 $x(t) + Q(x)(t) = y(t), t \in [0, T]$.

2) 定义 $\bar{Q}(y)(t) = Q(x)(t), \forall y \in C_r[0, T]$. 式中, $\forall y \in C_r[0, T]$ 是由条件(1)定义的唯一解. 则存在 $N_1 > 0$, 使

$$\|\bar{Q}(y)(t)\| \leq N_1(q + \int_0^t \|y(s)\|ds).$$

引理 3^[9] 设有一常数序列 $\{a_i\}_{i \geq 0} (a_i \geq 0)$, 收敛到零, 算子 $Q: C_r[0, T] \rightarrow C_r[0, T]$ 满足

$$\|Q_1(u)(t)\| \leq N_2(a_i + \int_0^t \|u(s)\|ds).$$

式中: $N_2 \geq 1$ 为常数; $C_r[0, T]$ 的 r 维向量取最大值范数, 设 $P(t)$ 为 $r \times r$ 维连续函数矩阵, 令 $P: C_r[0, T] \rightarrow C_r[0, T]$ 为

$$P(u)(t) = P(t)u(t).$$

如果 P 的谱半径小于 1, 则有

$$\lim_{n \rightarrow \infty} (P + Q_n)(P + Q_{n-1}) \cdots (P + Q_0)(u)(t) = 0. \quad (5)$$

定理 对于式(1)所示系统, 满足假设条件 1) ~ 5), 在学习律式(3), 式(4)的作用下, 若满足如下条件:

$$\rho[I + \lambda(t)C(t)B(t)\Gamma]^{-1} < 1, t \in [0, T], \quad (6)$$

则系统(1)在任意初始状态下有

$$\lim_{k \rightarrow \infty} y_k(t) = y_d(t).$$

证明 由式(1)及迭代学习律式(3), (4)可得

$$\begin{aligned} x_{k+1}(t) - x_k(t) &= \int_0^t (f(x_{k+1}(\tau)) - f(x_k(\tau))) d\tau + \\ &\quad \lambda(t)B(t)\Gamma e_{k+1}(t) - \int_0^t \frac{d(\lambda(t)B(t)\Gamma)}{dt} e_{k+1}(\tau) d\tau. \end{aligned} \quad (7)$$

上式两端取范数, 并根据引理 1 可得

$$\begin{aligned} \|x_{k+1}(t) - x_k(t)\| &\leq m_1 \|e_{k+1}(t)\| + m_2 \int_0^t \|e_{k+1}(\tau)\| d\tau + \\ &\quad L \int_0^t (m_1 \|e_{k+1}(\tau)\| + m_2 \int_0^\tau \|e_{k+1}(s)\| ds) e^{L(1-\tau)} d\tau \leq m_1 \|e_{k+1}(t)\| + M_1 \int_0^t \|e_{k+1}(\tau)\| d\tau. \end{aligned}$$

式中: $b = \sup_{t \in [0, T]} \|B(t)\|$, $h = \sup_{t \in [0, T]} \|\lambda(t) \cdot \Gamma\|$, $m_1 = bh$, $m_2 = \sup_{t \in [0, T]} \left\| \frac{d(\lambda(t) \cdot B(t) \cdot \Gamma)}{dt} \right\|$, $M_1 = m_2 + Lm_1 e^{L \cdot T} + TLM_2 e^{L \cdot T}$.

又因

$$\begin{aligned} e_{k+1}(t) - e_k(t) &= C(t)(x_k(t) - x_{k+1}(t)), \\ \text{所以有} \\ e_{k+1}(t) &= [I + \lambda(t) \cdot C(t) \cdot B(t) \cdot \Gamma]^{-1} e_k(t) + [I + \lambda(t) \cdot C(t) \cdot B(t) \cdot \Gamma]^{-1} C(t) \cdot \left[\int_0^t \frac{d(\lambda(t) \cdot B(t) \cdot \Gamma)}{dt} e_{k+1}(\tau) d\tau - \int_0^t (f(x_{k+1}(\tau)) - f(x_k(\tau))) d\tau \right]. \end{aligned} \quad (8)$$

假设

$$P(t) = [I + C(t)B(t)\Gamma]^{-1}, \quad (9)$$

定义算子

$$\begin{aligned} G_{k+1}(e_{k+1})(t) &= -[I + \lambda(t)C(t)B(t)\Gamma]^{-1} C(t) \cdot \\ &\quad \left[\int_0^t \frac{d(\lambda(t) \cdot B(\tau) \cdot \Gamma)}{d\tau} e_{k+1}(\tau) d\tau - \int_0^t (f(x_{k+1}(\tau)) - f(x_k(\tau))) d\tau \right], \end{aligned}$$

则式(8)可写成

$$e_{k+1}(t) + G_{k+1}(e_{k+1})(t) = P(t)e_k(t). \quad (10)$$

对算子 G_{k+1} 进行估计, 在式(9)两端取范数, 并将式(7)代入, 得

$$\begin{aligned} \|G_{k+1}(e_{k+1})(t)\| &\leq M_3 \int_0^t \|e_{k+1}(\tau)\| d\tau \\ &= M_3(q_1 + \int_0^t \|e_{k+1}(\tau)\| d\tau). \end{aligned} \quad (11)$$

$$\begin{aligned} \text{式中: } M_2 &= m_3 c L m_1 + m_3 c L M_1 T + m_3 c m_2, q_1 = 0, \\ M_3 &= \max(1, M_2), c = \sup_{t \in [0, T]} \|C(t)\|, m_3 = \\ &\quad \sup_{t \in [0, T]} \|[I + \lambda(t) \cdot C(t) \cdot B(t) \cdot \Gamma]^{-1}\|. \end{aligned}$$

假设 $e_{k+1}(t), e_k(t) \in C_r[0, T]$, 则

$$\|G_{k+1}(e_{k+1}(t) - G_{k+1}(e_k(t))\| \leq M_4 \int_0^t \|e_{k+1}(\tau) - e_k(\tau)\| d\tau. \quad (12)$$

式中 $M_4 = m_3 c m_2$.

由式(11)和式(12)可知, 满足引理 2 的两个条件, 所以由引理 2 可得存在 \bar{G}_{k+1} 使得式(10)变成下式:

$$e_{k+1}(t) + \bar{G}_{k+1}(Pe_k)(t) = P(t)e_k(t). \quad (13)$$

式中, \bar{G}_{k+1} 满足

$$\|\bar{G}_{k+1}(Pe_k)(t)\| \leq M_5(q_1 + \int_0^t \|P(\tau)e_k(\tau)\| d\tau). \quad (14)$$

其中 $M_5 > 0$.

定义算子 $Q_{k+1}: C_r[0, T] \rightarrow C_r[0, T]$ 为

$$Q_{k+1}(e_k(t))(t) = -\bar{G}_{k+1}(Pe_k)(t).$$

则存在 $M_6 \geq 1$, 使得 Q_{k+1} 也满足式(14), 即

$$\|G_{k+1}(e_k)(t)\| \leq \|M_6(q_1 + \int_0^t \|e_k(\tau)\| d\tau)\|. \quad (15)$$

则式(13)可写成

$$\begin{aligned} e_{k+1}(t) &= P(t)e_k(t) + Q_{k+1}(e_k)(t) \\ &= (P + Q_{k+1})(P + Q_k) \cdots (P + Q_1)(e_0)(t). \end{aligned}$$

因为 Q_{k+1} 满足式(15), 即满足引理 3 的条件, 所以由引理 3 和条件 P 的谱半径小于 1 可知, 有 $\lim_{k \rightarrow \infty} \|e_{k+1}(t)\| = 0$, 在 $t \in [0, T]$ 内, 故有

$$\lim_{k \rightarrow \infty} y_k(t) = y_d(t).$$

证毕.

3 平面二自由度机械臂数学模型

本文以加拿大 Quanser 公司研制的平面二自由度机械臂实验装置为被控对象, 讨论具体的学习律参数的设计问题. 其动力学方程如下:

$$\begin{cases} H_{11}\ddot{\theta}_1 + H_{12}\ddot{\theta}_2 - h\dot{\theta}_2^2 - 2h\dot{\theta}_1\dot{\theta}_2 = u_1, \\ H_{21}\ddot{\theta}_2 + H_{12}\ddot{\theta}_1 + h\dot{\theta}_1^2 = u_2. \end{cases}$$

式中各项参数可表示为

$$\begin{aligned} H_{11} &= m_1 l_{c1}^2 + I_1 + m_2 [l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos \theta_2] + I_2, \\ H_{22} &= m_2 l_{c2}^2 + I_2, H_{12} = m_2 l_1 l_{c2} \cos \theta_2 + m_2 l_{c2}^2 + I_2, \\ h &= m_2 l_{c2} \sin \theta_2. \end{aligned}$$

其中: $I_1 = I_2 = 0.1$, $l_1 = l_2 = 0.5$, $m_1 = m_2 = 1$; u_1, u_2 分别为加于两杆的关节力矩; θ_1, θ_2 为机械臂关节角位移量.

设 $x_1 = \theta_1, x_2 = \dot{\theta}_1 = \dot{x}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2 = \dot{x}_3$,

$$a_{24} = \frac{H_{22}h}{H_{11}H_{22} - H_{12}^2}, a_{22} = \frac{H_{12}h}{H_{11}H_{22} - H_{12}^2},$$

$$a_{224} = \frac{2H_{22}h}{H_{11}H_{22} - H_{12}^2}, b_{21} = \frac{H_{22}}{H_{11}H_{22} - H_{12}^2},$$

$$b_{22} = \frac{-H_{12}}{H_{11}H_{22} - H_{12}^2}, a_{42} = \frac{-H_{11}h}{H_{11}H_{22} - H_{12}^2},$$

$$a_{44} = \frac{-H_{12}h}{H_{11}H_{22} - H_{12}^2}, b_{41} = \frac{-H_{12}}{H_{11}H_{22} - H_{12}^2},$$

$$a_{424} = \frac{-2H_{12}h}{H_{11}H_{22} - H_{12}^2}, b_{42} = \frac{H_{11}}{H_{11}H_{22} - H_{12}^2}.$$

得到平面二自由度机械臂的状态空间方程:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ a_{24}x_4^2 + a_{22}x_2^2 + a_{224}x_2x_4 \\ x_4 \\ a_{42}x_2^2 + a_{44}x_4^2 + a_{424}x_2x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

输出为关节的角速度量时,系统输出方程为

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},$$

则系统输出矩阵和系统控制输入矩阵为

$$C(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B(t) = \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix}.$$

4 仿真研究

采用式(3), (4)形式的迭代学习律,选择增益矩阵 $\Gamma = (C(t)B(t))^{-1}$,指数型可变增益为 $\lambda(t) = e^m$. 则当 $t=0$ 时, $\lambda(0) = 1$. 所以,初始状态迭代学习形式如下:

$$x_{k+1}(0) = x_k(0) + B(0)\Gamma e_k(0).$$

设机械臂期望轨迹跟踪轨迹为圆,则对应 x 轴和 y 轴的期望轨迹为 $\sin(\pi t)$ 和 $\cos(\pi t)$,仿真时间区间为 $t \in [0, 2]$. 易知期望初始状态为 $x_0 = [0 \quad 3.14 \quad 1 \quad 0]$, 设系统初始状态任选为 $x(0) = [1 \quad 0 \quad 0 \quad 1]$, 并选取初始控制输入为 $u_1(t) = u_2(t) = 0$.

为对比分析,首先采用带初值学习的固定增益 D 型迭代学习控制,迭代学习控制律为

$$u_{k+1}(t) = u_k(t) + \Gamma \dot{e}_k(t).$$

迭代初态采用如下学习形式:

$$x_{k+1}(0) = x_k(0) + B(0)\Gamma e_k(0).$$

增益矩阵为

$$\Gamma = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

初态学习律为

$$x_{k+1}(0) = x_k(0) + B(0)\Gamma e_k(0).$$

仿真结果如图 2 所示.

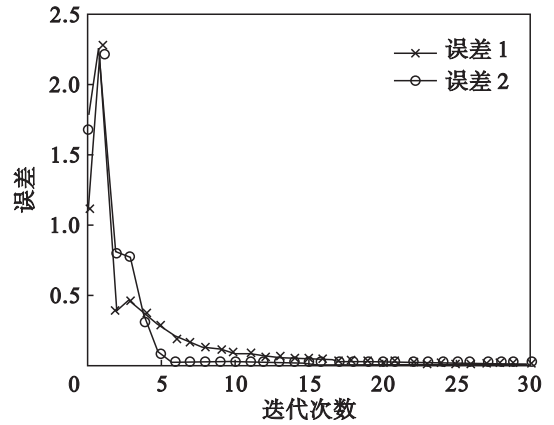


图 2 轨迹跟踪误差随迭代次数的变化曲线

Fig. 2 Trajectory tracking error curves along with the change of the number of iterations

由图 2 可见,带初值学习的固定增益 D 型迭代学习控制,需迭代到第 21 次才能完全跟踪上期望轨迹.

采用式(3),式(4)的迭代学习律,且

$$\lambda(t) = e^{0.8t}, \Gamma = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

当 $t=0$ 时,有

$$\rho[I + \lambda(t)C(t)B(t)\Gamma^{-1}] = 0.7696 < 1.$$

当 $t=2$ 时,有

$$\rho[I + \lambda(t)C(t)B(t)\Gamma^{-1}] = 0.4743 < 1.$$

因为 $\lambda(t)$ 是单调函数,且 $t=0$ 时刻和 $t=2$ 时刻均满足收敛条件,所以在整个区间 $t \in [0, 2]$ 内,所选取的增益系数 $\lambda(t)$ 和 Γ 都能满足式(6)所示的收敛条件. 仿真结果如图 3 所示.

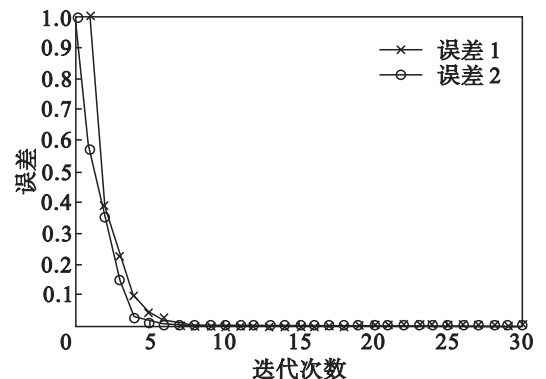


图 3 轨迹跟踪误差随迭代次数的变化曲线

Fig. 3 Trajectory tracking error curve along with the change of the number of iterations

由图 3 可见,带初值学习的变指数增益 D 型迭代学习控制,只需迭代到第 9 次就可完全跟踪上期望轨迹.

5 结 语

带初态学习的 D 型迭代学习控制算法,解决了系统初始状态误差所带来的输出轨迹不能很好跟踪期望轨迹的问题,但收敛速度表现不佳. 本文给出的带初态学习的指数变增益迭代学习律,既能消除初态偏移对迭代学习控制的影响,又能加快迭代学习的收敛速度. 以平面二自由度机械臂为被控对象的仿真实验验证了算法的有效性.

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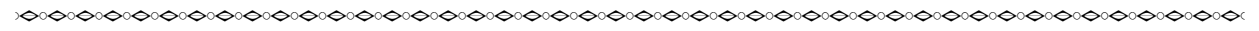
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受限的情况,给出了一个自适应律以保证系统在控制饱和时的稳定性. 为了使路由器中瞬时队列长度能在有限时间内收敛到期望值附近,给出了一个基于终端滑模控制的有限时间拥塞控制器. 仿真结果表明,本文所提出的控制器在不同的网络情境下不仅具有有限时间收敛的品质,而且具有良好的稳定性和鲁棒性.

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