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带有状态约束的非线性切换系统的 H_∞ 控制器设计

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摘　　要：研究了一类带有状态约束的下三角结构的非线性切换系统的 H_∞ 控制器设计问题，利用障碍 Lyapunov 函数方法处理状态约束问题. 针对该非线性切换系统利用反步设计技术来构造状态反馈控制器，在每一步的设计过程中，通过选取合适的障碍 Lyapunov 函数构造系统的共同障碍 Lyapunov 函数来处理状态约束条件，再经过一系列处理给出每步的虚拟控制器的具体表示形式，以此类推，从而设计出该非线性切换系统在任意切换条件下的 H_∞ 控制器. 最后通过算例进行数值仿真说明了所得结果的有效性.

关　键　词：切换系统；状态约束；反步；H_∞ 控制；仿真

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H_∞ Controller Design for a Class of Nonlinear Switched Systems with State Constraints

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Abstract: The H_∞ controller problem for a class of nonlinear switched systems with state constraints was investigated. Each subsystem is in lower triangular form. By employing the barrier Lyapunov function method, the issue of state constraints can be handled. Corresponding feedback controllers are constructed with the backstepping approach. To deal with the problem of state constraints, a barrier Lyapunov function is constructed to obtain the common barrier Lyapunov function for the whole system in every step, and a common virtual control is obtained step by step. A continuous state feedback H_∞ controller is provided for the nonlinear switched systems under arbitrary switchings. A numerical example is given to demonstrate the effectiveness of the main results.

Key words: switched systems; state constraints; backstepping; H_∞ control; simulation

切换系统作为一类混杂动态系统，在飞行器控制、机器人控制等领域均有着广泛的应用前景，对于切换系统的研究已取得了一定成果[1-5]. 最近，针对切换系统在状态受限条件下的控制问题引起了广泛关注[6-10]. 现有结果只解决了切换系统的一类跟踪问题，而 H_∞ 控制问题作为一类重要的鲁棒控制问题尚未涉及；同时前人在控制器设计过程中只假设虚拟控制器存在，并未给出其具体形式，而设计出每步虚拟控制器的具体形式对给出系统最终的状态反馈控制器十分重要.

本文研究了一类在任意切换条件下带有状态约束的非线性切换系统的全局 H_∞ 控制器设计问题，采用障碍 Lyapunov 函数方法处理状态受限问题，利用反步设计技术构造出每步的共同虚拟控制器，从而得到具体的状态反馈控制器的设计方案；最后通过数值仿真算例说明了所得结果的有效性.

1　系统描述与准备工作

考虑以下非线性切换系统：

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$$
\left.\begin{aligned}
&\dot z_1=f_{01,\sigma(t)}(z,x_1)+q_{01,\sigma(t)}(z,x_1)\boldsymbol{\omega};\\
&\dot z_2=f_{02,\sigma(t)}(z_2,x_1);\\
&\dot x_i=g_{i,\sigma(t)}(z,\bar{\boldsymbol{x}}_i)x_{i+1}+f_{i,\sigma(t)}(z,\bar{\boldsymbol{x}}_i)+\\
&\quad q_{i,\sigma(t)}(z,\bar{\boldsymbol{x}}_i)\boldsymbol{\omega},\\
&i=1,\cdots,r-1;\\
&\dot x_r=g_{r,\sigma(t)}(z,\boldsymbol{x})u_{\sigma(t)}+f_{r,\sigma(t)}(z,\boldsymbol{x})+\\
&\quad q_{r,\sigma(t)}(z,\boldsymbol{x})\boldsymbol{\omega};\\
&y=h_{\sigma(t)}(z,x_1)+d_{0,\sigma(t)}(z,\boldsymbol{x})\boldsymbol{\omega}.
\end{aligned}\right\}\quad(1)
$$

其中：$z,\boldsymbol{x},\boldsymbol{\omega}$ 分别为状态及扰动输入；$z=(z_1,z_2)^{\mathrm T}$，$(z_1,z_2)\in\mathbf{R}^{n_1+n_2}$，$n=n_1+n_2+r$；$\boldsymbol{x}=(x_1,x_2,\cdots,x_r)^{\mathrm T}$，$\bar{\boldsymbol{x}}_i=(x_1,x_2,\cdots,x_i)^{\mathrm T}$；$\sigma(t)\in M=\{1,\cdots,m\}$ 为切换信号；$u_{\sigma(t)}$ 为各子系统的控制输入；假设 $g_{i,\sigma(t)}(z,\overline{x_i})>0$，存在常数 $\gamma_{d0}>0$，有 $\|d_{0,k}(z,\boldsymbol{x})\|\leqslant\gamma_{d0}$. 另 $g_{i,k}(0,0)=0$，$f_{i,k}(0,0)=0$，$h_k(0,0)=0$，$d_{0,k}(0,0)=0$，$\forall k\in M$. 系统(1)的前两个方程为零动态方程.

本文研究在任意切换下的系统的 H_∞ 控制问题，给定标量 $\gamma>\gamma_{d0}\geqslant0$，设计控制器 u 使得：

1）当 $\boldsymbol{\omega}=0$ 时，闭环切换非线性系统在任意切换下为全局渐近稳定的；

2）对于任意初始条件 $(z_0,x_0)\in\mathbf{R}^n$ 及所有 $\boldsymbol{\omega}\in L_2$，存在 $W:\mathbf{R}^n\to\mathbf{R}_+$ 有 $W(0,0)=0$，使得

$$\int_0^\infty\|y(t)\|^2\mathrm{d}t\leqslant\gamma^2\int_0^\infty\|\omega(t)\|^2\mathrm{d}t+W(z_0,x_0).\quad(2)$$

2　主要结果

下面给出系统(1)在状态受限条件下 H_∞ 状态反馈控制器的设计过程.

引理 1[8]　对任意正整数 c,d 及函数 $\gamma(x,y)>0$，$|x|^c|y|^d\leqslant\dfrac{c}{c+d}\gamma(x,y)|x|^{c+d}+\dfrac{d}{c+d}\times\gamma^{-\frac{c}{d}}(x,y)|y|^{c+d}$.

定理 1　假设存在光滑正定的径向无界函数 $W_1(z_1)$，$W_2(z_2)$，有

① $\alpha_3\|z_2\|^2\leqslant W_2(z_2)$；

② $\dfrac{\partial W_1(z_1)}{\partial z_1}\dot z_1\leqslant-\alpha_1\|z_1\|^2+\gamma_0^2\|\boldsymbol{\omega}\|^2+\upsilon_1(z_2,x_1)$；

③ $\dfrac{\partial W_2(z_2)}{\partial z_2}f_{02,k}(z_2,\phi_0(z_2))\leqslant-\alpha_2W_2(z_2)$.

其中：取 $\alpha_1,\gamma_0,\alpha_2,\alpha_3$ 为正常数；$\phi_0(z_2)$ 为满足 $\phi_0(0)=0$ 的光滑函数；$\upsilon_1(z_2,x_1)$ 为正定径向无界函数，则在任意切换下该系统的全局 H_∞ 控制器可解.

证明　第一步：考虑系统(1)的 (z_1,z_2,x_1) 方程，取 $\eta_1=x_1-\phi_0(z_2)$，其中 $\phi_0(z_2)$ 取自③，

$$\dot\eta_1=g_{1,k}(z,x_1)x_2+F_{1,k}(z,x_1)+q_{1,k}(z,x_1)\omega,$$

其中，$F_{1,k}(z,x_1)=f_{1,k}(z,x_1)-\dfrac{\partial\phi_0}{\partial z_2}f_{02,k}(z_2,x_1)$.

由文献[8]，可知存在 $\hat\rho_{1,k}(z,\eta_1)$ 满足不等式

$$|F_{1,k}(z,x_1)|\leqslant(\|z_1\|+\|z_2\|+|\eta_1|)\hat\rho_{1,k}(z,\eta_1).$$

由引理 1[8] 可得

$$\frac{|\eta_1F_{1,k}(z,\eta_1)|}{b_1^2-\eta_1^2}\leqslant\frac{\alpha_1}{2}\|z_1\|^2+\frac12\|z_2\|^2+\frac{\eta_1^2\hat\rho_{1,k}^2}{2\alpha_1(b_1^2-\eta_1^2)^2}+\frac{\eta_1^2\hat\rho_{1,k}^2}{2(b_1^2-\eta_1^2)^2}+\frac{\eta_1^2\hat\rho_{1,k}}{b_1^2-\eta_1^2}.$$

在此，本文定义

$$V_1=W_1(z_1)+\frac{1}{\alpha_2}S_0(W_2(z_2))+\frac12\lg\Bigl(\frac{b_1^2}{b_1^2-\eta_1^2}\Bigr).$$

设 $S_j=\dot V_j(z,\overline{\eta_j})+\|y\|^2-\varepsilon_j^2\|\boldsymbol{\omega}\|^2$，$j=1,\cdots,r-1$，由条件①，②，③得到

$$\begin{aligned}S_1\leqslant&-\frac12\alpha_1\|z_1\|^2-\frac12\|z_2\|^2+\\&\frac{\eta_1}{b_1^2-\eta_1^2}\Bigl[\psi_1x_2+\frac{\eta_1\hat\rho_{1,k}^2}{2\alpha_1(b_1^2-\eta_1^2)}\Bigr]+\\&\frac{\eta_1}{b_1^2-\eta_1^2}\Bigl[\frac{\eta_1\hat\rho_{1,k}^2}{2(b_1^2-\eta_1^2)}+\eta_1\hat\rho_{1,k}^2+\frac{\eta_1q_1^2(z,x_1)}{4\tau_1^2(b_1^2-\eta_1^2)}\Bigr]+\\&\frac{\eta_1}{b_1^2-\eta_1^2}[H_{02}(z_2,\eta_1)+\delta(z_2)\chi^2(z_2,\eta_1)\eta_1+\eta_1].\end{aligned}$$

其中：$\chi^2(z_2,\eta_1)$，$\delta(z_2)$，$H_{01}(z_2)$，$H_{02}(z_2,\eta_1)$，$S_0(W_2(z_2))$，τ_1^2 的定义及计算过程见文献[8]，$\|q_1(z,x_1)\|=\max\limits_{k\in M}\{\|q_{1,k}(z,x_1)\|\}$，$\|\psi_1(z,x_1)\|=\max\limits_{1\leqslant i\leqslant r,k\in M}\{\|g_{i,k}(z,\overline{x_i})\|\}$.

取虚拟控制器为

$$\begin{aligned}\phi_1(z,\eta_1)=&-\frac{1}{\psi_1(z,x_1)}\Bigl[H_1(z_2,\eta_1)+\frac{\rho_1(z,\eta_1)\eta_1}{b_1^2-\eta_1^2}\Bigr]-\frac{1}{\psi_1(z,x_1)}\times\frac{q_1^2(z,x_1)\eta_1}{4\tau_1^2(b_1^2-\eta_1^2)^2}-\\&\frac{1}{\psi_1(z,x_1)}[\delta(z_2)\chi^2(z_2,\eta_1)\eta_1+\eta_1],\end{aligned}$$

即有

$$\begin{aligned}S_1\leqslant&-\Bigl(\alpha_1-\frac{\alpha_1}{2}\Bigr)\|z_1\|^2-\Bigl(\frac34-\frac12\Bigr)\|z_2\|^2-\\&\eta_1^2+\frac{\psi_1(z,x_1)\eta_1}{b_1^2-\eta_1^2}(x_2-\phi_1(z,\eta_1))\leqslant\\&-\beta_1(\|z\|^2+\eta_1^2)+\frac{\psi_1(z,x_1)\eta_1}{b_1^2-\eta_1^2}(x_2-\phi_1(z,\eta_1)),\end{aligned}$$

其中 $\beta_1=\min\{\frac{\alpha_1}{2},\frac{1}{4}\}$.

第 i 步($i=2,\cdots,r-1$)：利用递推法，假设第 $i-1$ 步中存在 $i-1$ 个正数 $\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_{i-1}$，满足 $\gamma_{d0}<\varepsilon_1<\varepsilon_2<\cdots<\gamma,\phi_0(z_2),\phi_j(z,\eta_1,\cdots,\eta_j)$, $\phi_0(0)=0,\phi_j(0)=0,j=1,\cdots,i-1$,则存在全局的坐标变换：

$$\eta_1=x_1-\phi_0(z_2),\eta_2=x_2-\phi_1(z,\eta_1),\cdots,$$

$$\eta_{i-1}=x_{i-1}-\phi_{i-2}(z,\eta_1,\cdots,\eta_{i-2}),$$

$$S_{i-1}\leqslant-\frac{\beta_1}{2^{i-2}}(\|z\|^2+\sum_{j=1}^{i-1}\eta_j^2)+\frac{\psi_1(z,x_1)\eta_{i-1}}{b_{i-1}^2-\eta_{i-1}^2}(x_i-\phi_{i-1}(z,\overline{\eta_{i-1}})).$$

其中 $V_i=\frac{1}{\tau_1^2}V_1(z,\eta_1)+\frac{1}{2}\sum_{j=2}^{i-1}\lg\frac{b_j^2}{b_j^2-\eta_j^2}$.

类似第一步,同理可得存在光滑的虚拟控制 $\phi_{i-1}(z,\eta_1,\cdots,\eta_{i-1})$ 使得 $S_i\leqslant-\frac{\beta_1}{2^{i-1}}(\|z\|^2+\sum_{j=1}^{i}\eta_j^2)$.

第 r 步：得到控制器

$$u=\phi_r(z,\eta_1,\cdots,\eta_r),\tag{3}$$

$$\dot{V}_r(z,\eta_1,\cdots,\eta_r)+\|y\|^2-\gamma^2\|\boldsymbol{\omega}\|^2\leqslant-\frac{\beta_1}{2^{r-1}}(\|z\|^2+\sum_{j=1}^{r}\eta_j^2).\tag{4}$$

其中 $V_r(z,\eta_1,\cdots,\eta_r)$为正定径向无界函数.

当 $\boldsymbol{\omega}\equiv0$ 时，由式(4)，有 $\dot{V}_r(z,\eta_1,\cdots,\eta_r)<0$, $\forall(z,\eta_1,\cdots,\eta_r)\neq0$, $\forall k\in M$, 得到渐近稳定性.

在任意初始状态下，将式(4)两端从 $t=0$ 到 $t=\infty$ 积分，有

$$0\geqslant\int_0^{t_1}(\dot{V}_r+\|y(s)\|^2-\gamma^2\|\omega(s)\|^2)\,\mathrm{d}s+\cdots+\int_{t_{j-1}}^{t_j}(\dot{V}_r+\|y(s)\|^2-\gamma^2\|\omega(s)\|^2)\,\mathrm{d}s+\cdots=\sum_{j=1}^{\infty}\int_{t_{j-1}}^{t_j}(\|y(s)\|^2-\gamma^2\|\omega(s)\|^2)\mathrm{d}s+\sum_{j=1}^{\infty}\int_{t_{j-1}}^{t_j}\dot{V}_j\mathrm{d}s.$$

即满足式(2)，定理证毕.

因此，本文中所定义的切换系统在控制器(3)作用下满足 H_∞ 控制问题的需要.

3 数值算例

考虑由如下切换子系统构造的切换系统：

子系统 1：	子系统 2：
$\dot{z}_1=z_2^2-z_1+\boldsymbol{\omega}$;	$\dot{z}_1=-z_1$;
$\dot{z}_2=x_1$;	$\dot{z}_2=z_1z_2+x_1$;
$\dot{x}_1=u+z_2^2+\boldsymbol{\omega}$;	$\dot{x}_1=u+\boldsymbol{\omega}$;
$y=z_2\sin x_1+\boldsymbol{\omega}$.	$y=z_2^2\cos x_1+\boldsymbol{\omega}$.

为满足定理 1 中的假设，选取 $\phi_0=-5z_2$, $b_1=0.4$.

$$\hat{\rho}_{1,1}=\frac{1}{4}[(5+z_1^5)(3+z_1^2)+(1+z_2^2)(1+z_1^2+2z_2^2)],$$

$$\hat{\rho}_{1,2}=\frac{1}{4}[(3+z_1^2+2z_2^2)(3+z_1^2)+2(1+z_2^2)+4z_1^2],$$

$$\rho_{1,1}=\frac{1}{\alpha_1}\hat{\rho}_{1,1}^2+\hat{\rho}_{1,1}^2+2\hat{\rho}_{1,1}(b_1^2-\eta_1^2),$$

$$\rho_{1,2}=\frac{1}{\alpha_1}\hat{\rho}_{1,2}^2+\hat{\rho}_{1,2}^2+2\hat{\rho}_{1,2}(b_1^2-\eta_1^2).$$

$\rho_1,\delta(z_2),\chi^2(z_2,\eta_1),\tau_1^2$ 按照定理 1 的计算公式给出，$|x_1|<b_1$，其他参数选取及计算公式与文献[7]中相同. 有

$$u=\phi_1(z,\eta_1)=-\left(\frac{\eta_1\rho_1}{2(b_1^2-\eta_1^2)}+\frac{\eta_1q_1^2}{4\tau_1^2(b_1^2-\eta_1^2)}\right)-\delta(z_2)\chi^2\eta_1-\eta_1x_1^2.$$

由定理 1 即可得到切换系统的全局 H_∞ 控制器. 图 1 及图 2 为在扰动 ω 设为 $\mathrm{e}^{-0.1t}$时对应的切换系统的仿真结果.

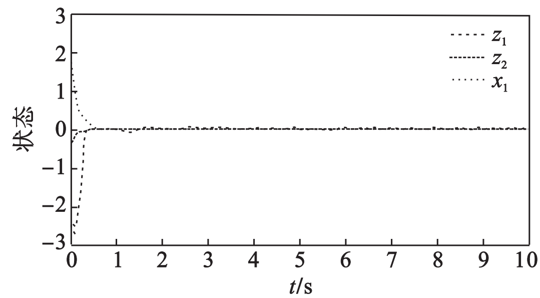

图 1 状态轨迹

Fig. 1 State trajectories

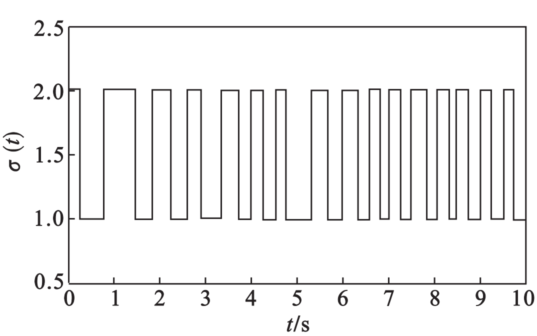

图 2 切换信号

Fig. 2 Switching signal

4 结 语

本文针对状态受限下的非线性切换系统设计

了 H_∞ 状态反馈控制器，并给出构造每步的虚拟控制器的具体方法，最后通过数值仿真说明了所得结果的有效性.

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