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随机多智能体系统一致增益问题分析

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摘　　要: 在随机多智能体系统一致增益函数为一常数函数不满足鲁棒性条件下，把现有文献中一致稳定性鲁棒性增益条件加以拓展，得到了保证系统趋于一致稳定性的新条件，分析了随机多智能体系统一致稳定性问题. 运用代数图理论和随机稳定性理论，把随机多智能体系统的一致稳定性问题通过系统变换转化为闭环随机多智能体系统状态差为变量的随机微分方程的稳定性问题;然后运用随机稳定性系统理论来分析闭环随机多智能体系统状态差系统是一致稳定的，从而得到了随机多智能体系统一致稳定性的条件. 最后通过实例来验证所提系统的可行性和有效性.

关 键 词: 随机控制; 多智能体系统; 增益函数; 一致性; 稳定性

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Consensus Gain Analysis of Stochastic Multi-agent System

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Abstract: The consensus stability of the stochastic multi-agent system was investigated when the consensus gain function did not satisfy the robustness condition of the consensus stability, and the robustness gain condition of consensus stability was broaden from the existing literature. By using the algebraic graph theory and the stochastic stability theory, the consensus stability problem of stochastic multi-agent system was transformed into the stability problem of the state-error closed-loop system. Next the stochastic state-error closed-loop multi-agent system was analyzed by the stochastic stability theory, then a new consensus stability condition of the stochastic multi-agent system was obtained. Finally, a simulation example was given to illustrate the feasibility and effectiveness of the proposed system.

Key words: stochastic control; multi-agent system; gain function; consensus; stability

近年来，随机多智能体系统分布一致稳定性控制问题由于其广泛的民用和军用而备受关注，其应用领域涉及交通控制、传感器网络控制[1]、移动机器人[2]、社会网络等等.

Olfati-Saber 等[3]通过代数图论把一阶多智能体系统的一致性问题等价为混杂系统的稳定性问题，并且通过随机矩阵的特性及其李雅普诺夫第二方法分析了一阶多智能体系统为一致稳定性. Li 等[4]在此基础上把一阶多智能体系统一致稳定性拓展到了一阶随机多智能体系统一致稳定性中，通过鞅收敛定理和代数图论结合李雅普诺夫泛函，把随机多智能体系统的无偏均方平均一致性问题转化为矩阵积的收敛问题，进而转化为一致性控制器增益的标量收敛问题. Cheng 等[5]通过确定二阶随机多智能体闭环系统的状态转移矩阵运用随机逼近的方法，把二阶随机多智能体系统的渐近无偏均方平均一致性问题转化为随机微分方程的收敛问题来处理，从而完成二阶随机多智能体系统一致稳定性的分析. Miao 等[6]对高阶随机多智能体系统一致稳定性的证明进行了拓展性探索，先是通过系统状态转变，把状态随机微分方程转化为以状态差为变量的随机微分方程，

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然后利用李雅普诺夫定理和劳斯－赫维茨稳定性判据分析系统状态差的一致稳定性. Amelina 等[7]使用定常步长的随机逼近算法代替逐渐下降至零步长的随机逼近算法,结合平均模型方法分析了非线性随机多智能体系统的一致稳定性.

以上文献一般都假设随机多智能体系统一致增益函数$c(t)$满足鲁棒性条件:$\int_0^{\infty} c^2(t)\mathrm{d}t < \infty$. 本文假设随机多智能体系统一致增益函数 $c(t)$ 为一常数函数不满足鲁棒性条件下,分析随机多智能体系统一致稳定性,得到了随机多智能体系统一致稳定性的条件.

1 问题描述

本文讨论由 N 个智能体组成的随机多智能体系统,每个智能体为同构智能体,其中第 i 个智能体描述为

$$\dot{x}_i(t) = u_i(t), \qquad i = 1,\cdots,N, \tag{1}$$

其中 $x_i(t) \in \mathbf{R}$ 和 $u_i(t) \in \mathbf{R}$ 分别表示第 i 个智能体状态变量和控制变量. 第 i 个智能体能够从邻居智能体中获得带随机通信干扰的信息:

$$y_{ji}(t) = x_j(t) + (x_i(t) - \bar{x}(t))n_{ji}(t), j \in N_i. \tag{2}$$

其中:$y_{ji}(t)$为第 i 个智能体对第 j 个智能体状态的测量信息变量;$\bar{x}(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$;$\{n_{ji}(t), 1 \leqslant i,j \leqslant N\}$ 是定义在概率空间(Ω, F, P)中的标准独立高斯白噪声.

为了达到随机多智能体系统的一致性,本文设计了如下的分布式控制律:

$$u_i(t) = c(t)\sum_{j\in N_i} a_{ij}(y_{ji}(t) - x_i(t)). \tag{3}$$

这里 $c(t)$ 是一致增益函数. 先定义 $J = (1/N).\mathbf{1}\mathbf{1}^{\mathrm{T}}, \mathbf{1} = [1,1,\cdots,1]^{\mathrm{T}}$ 为 N 维向量,同时定义状态差 $\boldsymbol{\delta}(t) = (\boldsymbol{I}_N - \boldsymbol{J})\boldsymbol{X}(t)$,然后把分布式控制律(3)代入随机多智能体系统(1)和(2)中,得

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= u_i(t) = c(t)\sum_{j\in N_i} a_{ij}(y_{ij}(t) - x_i(t)) = \\ &- c(t)\Big[\Big(\sum_{j\in N_i} a_{ij}\Big)x_i(t) - \sum_{j\in N_i}(a_{ij}x_j(t))\Big] + \\ &c(t)\mathrm{deg_{in}}(i)(x_i(t) - \bar{x}(t))n_{ji}(t).\end{aligned} \tag{4}$$

其中:$\mathrm{deg_{in}}(i) = \sum_{j=1}^{N} a_{ij}$ 为智能体 i 为顶点的入度;噪声过程 $\{n_{ji}(t), 1 \leqslant i, j \leqslant N\}$ 满足 $\int_0^t n_{ji}(s)\mathrm{d}s = \omega_{ij}(t), t \geqslant 0$,其中 $\{\omega_{ij}(t), 1 \leqslant i,j \leqslant N\}$ 是独立布朗运动过程. 那么随机多智能体系统就变为

$$\mathrm{d}\boldsymbol{X}(t) = -c(t)\boldsymbol{L}_G\boldsymbol{X}(t)\mathrm{d}t - c(t)\boldsymbol{D}_G\boldsymbol{\delta}(t)\mathrm{d}\boldsymbol{W}(t). \tag{5}$$

其中:$\boldsymbol{X}(t) = [x_1(t), \cdots, x_N(t)]^{\mathrm{T}}$;$\boldsymbol{D}_G = \mathrm{diag}(\mathrm{deg_{in}}(1),\cdots,\mathrm{deg_{in}}(N))$;$\boldsymbol{L}_G = \boldsymbol{D}_G - \boldsymbol{A}_G$ 为智能体系统图 G 的拉普拉斯矩阵;$\boldsymbol{W}(t) = [\boldsymbol{\omega}_1^{\mathrm{T}}(t), \cdots, \boldsymbol{\omega}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\boldsymbol{\omega}_i(t) = [\omega_{1i}(t), \cdots, \omega_{Ni}(t)]^{\mathrm{T}}$.

由于 $\boldsymbol{I}_N - \boldsymbol{J}$ 是单元对称矩阵,故$(\boldsymbol{I}_N - \boldsymbol{J})^{\mathrm{T}} = \boldsymbol{I}_N - \boldsymbol{J}, (\boldsymbol{I}_N - \boldsymbol{J})^{\mathrm{T}}(\boldsymbol{I}_N - \boldsymbol{J}) = \boldsymbol{I}_N - \boldsymbol{J}$,于是

$$\begin{aligned}\frac{\partial\boldsymbol{\delta}(t)}{\partial t} &= \frac{\partial((\boldsymbol{I}_N - \boldsymbol{J})\boldsymbol{X}(t))}{\partial t} = \\ &- c(t)\boldsymbol{L}_G\boldsymbol{X}(t) + c(t)\boldsymbol{J}\boldsymbol{L}_G\boldsymbol{X}(t) - \\ &c(t)(\boldsymbol{I}_N - \boldsymbol{J})\boldsymbol{D}_G\boldsymbol{\delta}(t, \boldsymbol{X}(t))\boldsymbol{W}(t),\end{aligned} \tag{6}$$

即

$$\begin{aligned}\mathrm{d}\boldsymbol{\delta}(t) = &-c(t)\boldsymbol{L}_G\boldsymbol{\delta}(t)\mathrm{d}t - c(t)(\boldsymbol{I}_N - \boldsymbol{J})\boldsymbol{D}_G\boldsymbol{\delta}(t)\cdot \\ &\mathrm{d}\boldsymbol{W}(t).\end{aligned} \tag{7}$$

2 随机多智能体系统一致稳定性

假设 1 随机多智能体系统连接拓扑图 G 为强连通图和平衡图.

引理 1[8] 对于随机多智能体系统(7)来说,如果存在一个正定函数 $V(\boldsymbol{\delta}(t), t)$,使得 $\mathscr{L}V(\boldsymbol{\delta}(t), t) < 0$对于所有的$\boldsymbol{\delta}(t)$都成立,那么随机多智能体系统(7)就是随机稳定的.

定理 1 在假设 1 成立的条件下,运用分布式控制律(3)控制随机多智能体系统(1)和(2),如果一致增益函数 $c(t)$满足

$$0 < c(t) < \frac{4\lambda_2((\boldsymbol{L}_G + \boldsymbol{L}_G^{\mathrm{T}})/2)}{\mathrm{tr}(\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N - \boldsymbol{J})\boldsymbol{D}_G)}, \tag{8}$$

则随机多智能体闭环系统(7)一致稳定,分布式控制律(3)为一致稳定控制律.

证明 构造李雅普诺夫函数如下:

$$V(t, \boldsymbol{\delta}(t)) = \boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{\delta}(t). \tag{9}$$

李雅普诺夫函数 $V(t, \boldsymbol{\delta}(t))$对 t 求偏导可得

$$\begin{aligned}V_t(t, \boldsymbol{\delta}(t)) = &-c(t)\boldsymbol{\delta}^{\mathrm{T}}(t)(\boldsymbol{L}_G^{\mathrm{T}} + \boldsymbol{L}_G)\boldsymbol{\delta}(t) - c(t)\times \\ &\boldsymbol{W}^{\mathrm{T}}(t)\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G\boldsymbol{\delta}(t) - c(t)\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G\boldsymbol{\delta}(t)\boldsymbol{W}(t).\end{aligned} \tag{10}$$

李雅普诺夫函数 $V(t, \boldsymbol{\delta}(t))$对$\boldsymbol{\delta}(t)$求偏导可得

$$V_\delta(t, \boldsymbol{\delta}(t)) = \frac{\partial V(t, \boldsymbol{\delta}(t))}{\partial\boldsymbol{\delta}(t)} = \frac{\partial(\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{\delta}(t))}{\partial\boldsymbol{\delta}(t)} = 2\boldsymbol{\delta}^{\mathrm{T}}(t).$$

李雅普诺夫函数 $V(t, \boldsymbol{\delta}(t))$对$\boldsymbol{\delta}(t)$求二阶偏导可得

$$V_{\delta\delta}(t,\boldsymbol{\delta}(t)) = \frac{\partial V(t,\boldsymbol{\delta}(t))}{\partial\boldsymbol{\delta}(t)\partial\boldsymbol{\delta}(t)} = 2 .$$

注意到状态差 $\delta(t)$ 是随机微分方程(7)的解，$V(t,\boldsymbol{\delta}(t))$ 是连续可偏微分的正定函数. 根据 Mao[9] 定义的 Itô 微分算子 $\mathscr{L}V:\mathbf{R}^n\times\mathbf{R}_+\to\mathbf{R}$，$V(t,\boldsymbol{\delta}(t))$ 有一随机微分

$$\mathscr{L}V(t,\boldsymbol{\delta}(t)) = -c(t)\boldsymbol{\delta}^{\mathrm{T}}(t)(\boldsymbol{L}_G^{\mathrm{T}}+3\boldsymbol{L}_G)\boldsymbol{\delta}(t) + c(t)\cdot\boldsymbol{W}^{\mathrm{T}}(t)\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G\boldsymbol{\delta}(t) - c(t)\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G\boldsymbol{\delta}(t)\cdot\boldsymbol{W}(t) + c^2(t)\,\mathrm{tr}(\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_G\boldsymbol{\delta}(t)) . \tag{11}$$

考虑到由 $\boldsymbol{L}_G$ 和 $\boldsymbol{L}_G^{\mathrm{T}}$ 所生成的二次型：

$$\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{L}_G\boldsymbol{\delta}(t)=\sum_{i,j=1}^{N}\boldsymbol{L}_{ij}\boldsymbol{\delta}_i^{\mathrm{T}}(t)\boldsymbol{\delta}_j(t) = \boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{L}_G^{\mathrm{T}}\boldsymbol{\delta}(t) . \tag{12}$$

这里 $\boldsymbol{L}_{ij}$ 是拉普拉斯矩阵 $\boldsymbol{L}_G$ 的元素，并且 $(\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}})/2$ 是对称的. 所以

$$E[\mathscr{L}V(t,\boldsymbol{\delta}(t))] = -4c(t)E[\boldsymbol{\delta}^{\mathrm{T}}(t)\frac{\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}}}{2}\boldsymbol{\delta}(t)] + c^2(t)\,\mathrm{tr}(\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_G\boldsymbol{\delta}(t)) . \tag{13}$$

因为 $E[\mathrm{d}\omega_{ji}(t)] = 0$，并且参考文献 [10]，对式(11)两边同时取数学期望有

$$E[\mathscr{L}V(t,\boldsymbol{\delta}(t))] = -c(t)E[\boldsymbol{\delta}^{\mathrm{T}}(t)(\boldsymbol{L}_G^{\mathrm{T}}+3\boldsymbol{L}_G)\cdot\boldsymbol{\delta}(t)] + c^2(t)\,\mathrm{tr}(\boldsymbol{\delta}^{\mathrm{T}}(t)\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_G\boldsymbol{\delta}(t)) .$$

如果系统连接拓扑图 G 是连接图，就有 $\lambda_2[(\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}})/2]>0$，称之为 $(\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}})/2$ 的最小非零特征值. 根据 Courant – Fisher 定理[11]，可得对称阵特性[12]：

$$\min_{\boldsymbol{\delta}(t)\neq 0,\mathbf{1}^{\mathrm{T}}\perp\boldsymbol{\delta}(t)}\frac{\boldsymbol{\delta}(t)^{\mathrm{T}}(\frac{\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}}}{2})\boldsymbol{\delta}(t)}{\|\boldsymbol{\delta}(t)^2\|}\geqslant\lambda_2(\frac{\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}}}{2}) .$$

于是就有

$$E[\mathscr{L}V(t,\boldsymbol{\delta}(t))]\leqslant -c(t)[4\lambda_2(\frac{\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}}}{2})\times\|\boldsymbol{\delta}(t)\|^2 c(t)\times\mathrm{tr}(\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_G)\|\boldsymbol{\delta}(t)\|^2].$$

适当地选取一致增益 $c(t)>0$，使它满足 $E[\mathscr{L}V(t,\boldsymbol{\delta}(t))]<0$，也就是

$$0<c(t)<\frac{4\lambda_2((\boldsymbol{L}_G+\boldsymbol{L}_G^{\mathrm{T}})/2)}{\mathrm{tr}(\boldsymbol{D}_G^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_G)}.$$

根据引理 1，这就证明了随机多智能体系统(7)是随机一致稳定的，分布式控制律(3)为一致稳定控制律.

3　仿真研究和分析

本节将给出实例来验证随机多智能体系统(4)的一致稳定性. 考虑由 3 个智能体构成的随机多智能体系统(见图 1)，该无向拓扑图 $V_G=\{1,2,3\}$，$E_G=\{(1,2),(2,3),(3,1)\}$，邻接矩阵为

$$\boldsymbol{A}_G=\begin{bmatrix}0 & 1\\ 1 & 0\\ 1 & 1\end{bmatrix}.$$

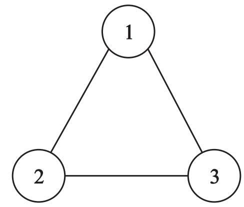

图 1　多智能体系统网络拓扑

Fig. 1　Network topology of the multi-agent system

选取测量噪声的强度为 $\sigma_{\omega_{12}}(t)=\sigma_{\omega_{23}}(t)=\sigma_{\omega_{31}}(t)=0.4$，选取各智能体初始状态为 $x_1(0)=-2$，$x_2(0)=-4$，$x_3(0)=6$，可以计算 $\mathrm{tr}(\boldsymbol{D}_{G_1}^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_{G_1})=4$，接着得到 $\dfrac{4\lambda_2((\boldsymbol{L}_{G_1}+\boldsymbol{L}_{G_1}^{\mathrm{T}})/2)}{\mathrm{tr}(\boldsymbol{D}_{G_1}^{\mathrm{T}}(\boldsymbol{I}_N-\boldsymbol{J})\boldsymbol{D}_{G_1})}=1.5$，这里选取一致性增益函数为常数函数 $c(t)=1$.

图 2，图 3 分别为无噪声干扰和有噪声干扰时的各状态曲线图.

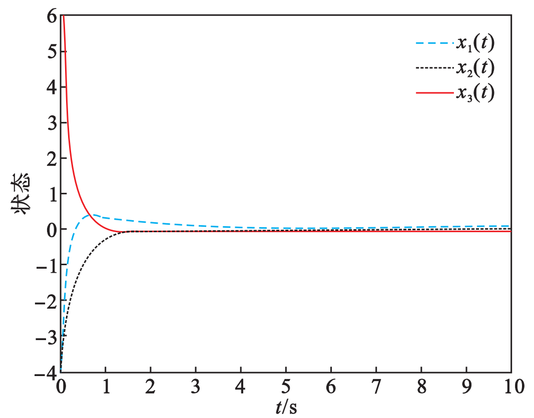

图 2　当 $c(t)=1$ 无噪声干扰时各状态曲线图

Fig. 2　Curves of states when $c(t)=1$ with no noise

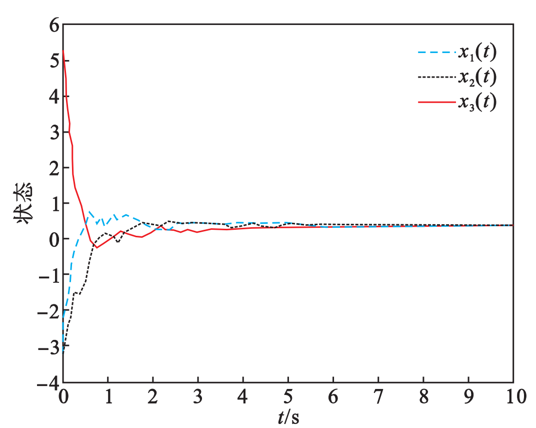

图 3　当 $c(t)=1$ 时有噪声干扰时各状态曲线图

Fig. 3　Curves of states when $c(t)=1$ with noise

从图2和图3可以看出,在分布式控制律(3)作用下,随机多智能体闭环系统(4)各个状态变量随着时间变量渐近达到一致,一致值为所有智能体初值的平均值,故而证明了随机多智能体闭环系统(4)是一致稳定的,分布式控制律(3)为一致稳定性控制律.

4 结 论

本文研究了带通信噪声的随机多智能体系统一致稳定性问题,当随机多智能体系统的一致性增益函数 $c(t)$ 为一常数函数时,尽管不满足鲁棒性条件:$\int_0^\infty c^2(t)\mathrm{d}t < \infty$,但是随机多智能体系统仍然能达到一致稳定性,并且得到了新的随机多智能体系统一致稳定性条件.

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